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A Mathematical Model Regarding Change in Preferences of Refugee Settlements

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Cover Page Footnote

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A Mathematical Model Regarding Change in Preferences of Refugee Settlements

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Abstract. Where cultures meet, there is bound to be conflict to some extent. This especially applies in the case of refugees grouped together when seeking asylum, with different styles of life, socialization, and conflict resolution meeting in one place. This paper focuses specially on three types of conflict resolution(negotiation, mediation, and arbitration) and constructs a differential equation model to study how the interactions between populations cause the number of people following each resolution method to shift. It was found that when there is no existing outside authority or environmental bias towards a resolution method, the method with the greatest number of followers will also be the one to take over the final population. However, in the presence of an outside force promoting or discouraging certain methods, although some groups will be given advantages over others, the final outcome is also still partially under the influence of the initial population. Outside of stable equilibria representing situations where one method ends up taking over the entire population, we also found certain unstable equilibria that carry key information about the basins of attraction of the stable equilibria.

1 Introduction

Refugees are men, women, and children who have crossed borders to seek safety in another country due to wars, political unrest, etc. Most refugees considered in this model are ones who cannot return home because of continuous conflict, wars, and persecution. In fact, globally speaking, there were nearly 26 million refugees by the end of 2020 [2]. Under these circumstances, it is important to provide refugees with safe and stable settlements. However, the diverse conditions and preferences of different refugees complicate this effort. In this paper, we develop a mathematical model that tracks and predicts refugees' changing preferences in dispute resolution strategy based on the ratio of the population of the population preferring each strategy. In other words, we assume a large population of refugees prefer one of three methods differently in each trial.

Mathematics Subject Classification. 92D95,97M10
Keywords. conflict resolution, differential equations, modeling

Generally, outside organizations tend to use three different methods to settle conflicts between groups: *negotiation*, *arbitration*, and *mediation*. In addition to outside interference, refugees from different areas have distinct cultural backgrounds and preferences for living and seek to incorporate their traditions into the settling process. The clash of diverse ideas results in disagreements among the refugees, outside organizations, and others involved in refugee settlements, requiring a solution to address the dissent. To better understand the issue, we next give the definitions of the three approaches. *Negotiation* is the process by which two stakeholders in a conflict or dispute reach a mutually agreed upon solution through discussion. *Mediation* is a process in which a neutral third party helps the parties reach an agreement without making a final decision. The mediator manages the communication process between the parties fairly, honestly, and impartially. *Arbitration* involves an arbitrator, chosen by both interested parties, who hears the case presented by each party and makes the final decision.

To explore the interaction of these resolution methods, in Section 2 we create and adjust a mathematical model to represent the population flow. In Section 3, we present and analyze the results yielded from the models, focusing on the equilibrium points. In Section 4, we apply the results of the model back into the original context of refugee settlements in the concluding discussion.

2 Method

To develop our model, we make the three following assumptions. (1) We assume that all refugees have a preference for either negotiation, mediation, or arbitration, with none undecided; (2) we also assume that the chance of a refugee switching to a preference linearly depends on the ratio of the population having that preference and the interpersonal interactions between the two groups of preferences (Theoretically, if the number of interactions double, the number of refugees that change preferences would also double, so we used a linear dependence model); (3) there is some outside force fueling the settlement that may or may not have its own preference, as reflected in the constants in the model.

2.1 Mathematical model

After setting our assumptions, we proceed to develop an initial model. The three states in 1 represent the preferences of a refugee in the population between mediation, arbitration, or negotiation. Refugees can switch between groups as their preferences change over time, guided by interactions with people with other preferences and forceful pushes by external organizations. Proportionality constants model the rate of movement from a group to another group. For example, k_{MA} represents the rate of movement from the mediation group to the arbitration group while k_{AM} represents the rate of movement from the arbitration group to the mediation group, and likewise for the other constants.

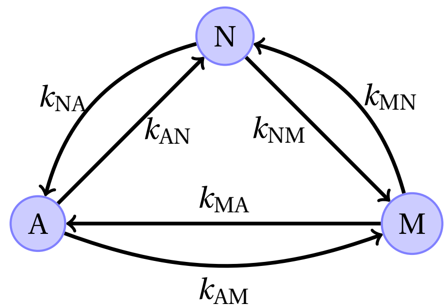


Figure 1: Diagram of population flow, with the coefficients representing the rate of conversion between groups with different preferences, where N represents the group preferring negotiation as a conflict resolution method, A represents those preferring arbitration, and M represents those preferring mediation.

From the diagram in Fig. 1, the following system is developed:

$$\begin{aligned} \frac{d\mathrm{N}}{dt} &= k_{\mathrm{AN}}\mathrm{AN} + k_{\mathrm{MN}}\mathrm{MN} - k_{\mathrm{NA}}\mathrm{NA} - k_{\mathrm{NM}}\mathrm{NM} \\ \frac{d\mathrm{A}}{dt} &= k_{\mathrm{NA}}\mathrm{NA} + k_{\mathrm{MA}}\mathrm{MA} - k_{\mathrm{AN}}\mathrm{AN} - k_{\mathrm{AM}}\mathrm{AM} \\ \frac{d\mathrm{M}}{dt} &= k_{\mathrm{NM}}\mathrm{NM} + k_{\mathrm{AM}}\mathrm{AM} - k_{\mathrm{MN}}\mathrm{MN} - k_{\mathrm{MA}}\mathrm{MA}. \end{aligned} \tag{2.1}$$

Here, N represents the number of people with a preference for negotiation, A represents the number with a preference for arbitration, and M represents the number with a preference for mediation. The interactions between two groups of people are modeled by the product of the number of people in those groups: for example, the interactions between groups A and N are modeled by AN. The instantaneous change in each of the groups is simply the difference between the people coming in from each of the other groups and the people leaving to each of the other groups, with the constants controlling this change. When these equations are summed, these terms cancel out and to get zero, showing that the net change in the population is zero, validating one of our prior assumptions.

However, this model does have some weaknesses. Since the constants are fixed, they determine in which direction people move in the diagram. For example, if k_{AN} is greater than k_{NA}, there is a net change only from arbitration to negotiation but not from negotiation to arbitration. This is not realistic; consider a situation in which almost all members of a population have a preference for arbitration except for a few favoring negotiation. Then, according to this model, everyone will eventually change their preference to negotiation. Instead, what happens in the real world is that people

generally yield to peer pressure and a standardized norm or conformity, following the majority of the population. What should happen in a case where almost all members of a population have a specific preference is that the population should gradually change until everyone has that preference. Therefore, the rates of change between the groups depend on another condition: the draw of the group being moved to. This is represented by the ratio of the number of people in that group to the entire population. Hence, we revise the model as follows:

$$\begin{aligned}\frac{d\mathrm{N}}{dt} &= k_{\mathrm{AN}}\tfrac{\mathrm{N}}{\mathrm{P}}\mathrm{AN} + k_{\mathrm{MN}}\tfrac{\mathrm{N}}{\mathrm{P}}\mathrm{MN} - k_{\mathrm{NA}}\tfrac{\mathrm{A}}{\mathrm{P}}\mathrm{NA} - k_{\mathrm{NM}}\tfrac{\mathrm{M}}{\mathrm{P}}\mathrm{NM}\\ \frac{d\mathrm{A}}{dt} &= k_{\mathrm{NA}}\tfrac{\mathrm{A}}{\mathrm{P}}\mathrm{NA} + k_{\mathrm{MA}}\tfrac{\mathrm{A}}{\mathrm{P}}\mathrm{MA} - k_{\mathrm{AN}}\tfrac{\mathrm{N}}{\mathrm{P}}\mathrm{AN} - k_{\mathrm{AM}}\tfrac{\mathrm{M}}{\mathrm{P}}\mathrm{AM} \qquad (2.2)\\ \frac{d\mathrm{M}}{dt} &= k_{\mathrm{NM}}\tfrac{\mathrm{M}}{\mathrm{P}}\mathrm{NM} + k_{\mathrm{AM}}\tfrac{\mathrm{M}}{\mathrm{P}}\mathrm{AM} - k_{\mathrm{MN}}\tfrac{\mathrm{N}}{\mathrm{P}}\mathrm{MN} - k_{\mathrm{MA}}\tfrac{\mathrm{A}}{\mathrm{P}}\mathrm{MA}\end{aligned}$$

where $\mathrm{P} = \mathrm{N} + \mathrm{A} + \mathrm{M}$.

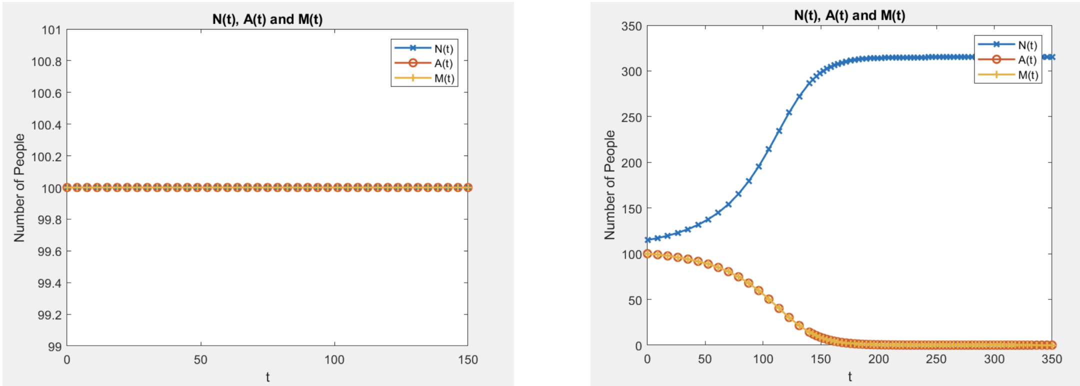
In this new model, the rate of moving from A to N is also proportional to the size of N relative to the population. This fixes many of the problems of the previous model and more closely matches real-world behavior. The sum of the three equations is still zero, meaning the total population does not change. The drawback of this model is that it is more difficult to analyze due to its complex nature, though we consider in the following section particular situations where the analysis is possible.

3 Results

3.1 All rate constants are equal

While we cannot directly find solutions to this general revised model, we can analyze the behavior with various initial conditions and parameter values. In some specific cases, as shown in Fig. 2a, the initial sizes and parameter values of the three groups are the same, so the number of people moving out of one group is equal to the number of people moving in, causing no overall change.

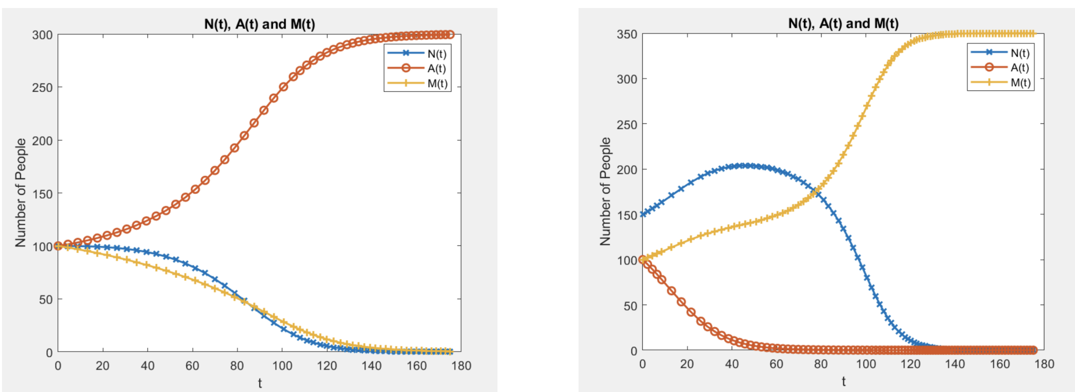
In Fig. 2b, all constants are 0.0002, but N has a slight initial majority with 115 individuals while the other groups have 100 individuals initially. In this case, the negotiation preference takes the eventual majority, directly as a result of more refugees initially favoring it.



(a) N(0) = A(0) = M(0) = 100

(b) N(0) = 115 and A(0) = M(0) = 100

Figure 2: Populations movement with the same rate constants (0.0002)



(a) $k_{NA} = 0.0003$, $k_{NM} = 0.0002$, $k_{AN} = 0.0002$, $k_{AM} = 0.0002$, $k_{MN} = 0.0003$, and $k_{MA} = 0.0002$, $A(0) = N(0) = M(0) = 100$

(b) $k_{NA} = 0.0004$, $k_{NM} = 0.0004$, $k_{AN} = 0.0005$, $k_{AM} = 0.0005$, $k_{MN} = 0.00025$ and $k_{MA} = 0.0002$, $A(0) = M(0) = 100, N(0) = 150$

Figure 3: Populations movement with differing rate constants

3.2 Rate constants are different

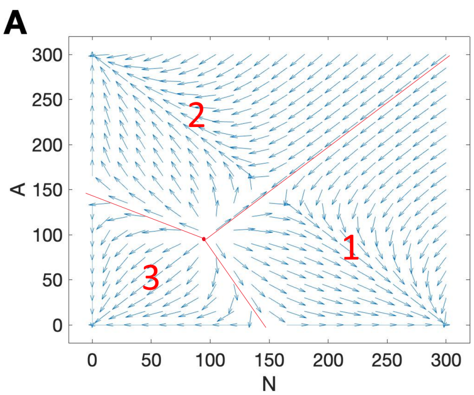
Fig. 3a shows a case in which all three states start with 100 individuals and the constants are $k_{NA} = 0.0003$, $k_{NM} = 0.0002$, $k_{AN} = 0.0002$, $k_{AM} = 0.0002$, $k_{MN} = 0.0003$, and $k_{MA} = 0.0002$. Although the initial values for A, N, and M are all the same, in this case, the constants determine which group wins out, allowing the arbitration preference to take the eventual majority. This is analogous to external factors such as non-governmental organizations directly impacting the change in refugee preferences.

Fig. 3b shows a case in which the initial majority favors negotiation, yet the constants are chosen such that the eventual majority does not favor negotiation, but instead mediation. The constants in this case are

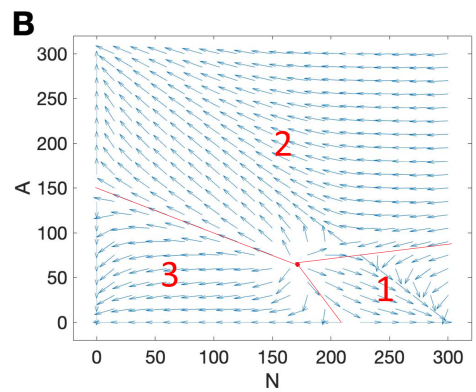
$k_{NA} = 0.0004$, $k_{NM} = 0.0004$, $k_{AN} = 0.0005$, $k_{AM} = 0.0005$, $k_{MN} = 0.00025$, and $k_{MA} = 0.0002$, with 150 individuals favoring negotiation initially while 100 favor arbitration and mediation initially. The constants here overrule the initial majority since the relatively low amounts of flow into the negotiation and arbitration states lead to the majority being the mediation preference by the end.

3.3 Direction fields

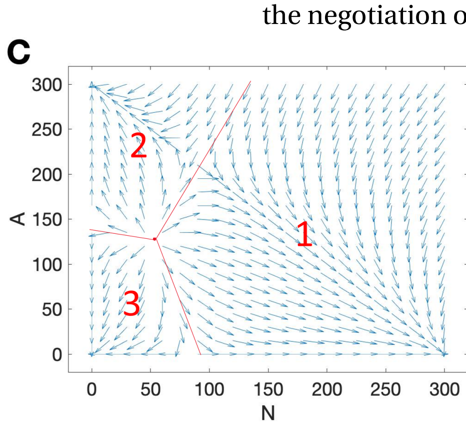
To look at the big picture for the dependence of the final result on the initial populations, we plot direction fields with three different rate constants in Fig. 4.



(a) All constants are set to 0.0002



(b) k_{NA} and k_{NM} are raised to 0.0005 to increase the negotiation outflow



(c) k_{AN} and k_{MN} are raised to 0.0005 to increase the negotiation inflow

Figure 4: Direction fields with varying rate constants

In all the rate constants we have explored in Fig. 4, the time series generally approach one of the stable equilibria $(P, 0)$, $(0, P)$, and $(0, 0)$, which correspond to $(N, A, M) = (P, 0, 0)$,

$(N, A, M) = (0, P, 0)$, and $(N, A, M) = (0, 0, P)$. In other words, all members of the community eventually come to an agreement for a singular resolution method except in the extremely rare case of an unstable equilibrium, for if the initial condition is just slightly outside these central unstable equilibria, the state tends to move towards a stable equilibrium (one of the endpoints of the graph). The basin of attraction for each of the equilibria depends on the rate constants as shown in Fig. 4, where Region 1 is the basin of attraction of (P, 0), Region 2 is the basin of attraction of (0, P), and Region 3 is the basin of attraction of (0, 0).

Figures 4b and 4c with reference to Figure 4a indicate that rate constants do not hold all of the power in determining the final outcome and show the influence of the initial population. When the initial population of any given group is sufficiently large, it can still take over the full population even with a larger outflow than inflow, as exemplified by Region 1 in Fig. 4b and Regions 2 and 3 in Fig. 4c.

From Fig. 4, we can observe two properties: (1) As inflow increases, the size of the respective basin increases, and, (2) the unstable equilibrium is at the intersection of all three basins. That means the equilibrium holds key information about the basins of attraction, making it useful to know where it is located.

So the important question is: when do these unstable equilibria occur? In Fig. 5, we plot how the group sizes and unstable equilibrium depend on the constant rates. To take an example, as k_{NA} increases, a smaller initial population A is needed to take over all of P.

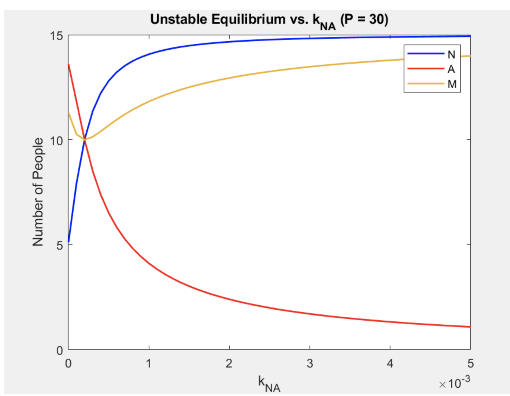


Figure 5: Movement of the unstable equilibria based on the value of k_{NA}.

4 Conclusion and discussion

Applying the equilibrium points from the model to the original context of the refugees, it follows that the populations of people preferring either negotiation, arbitration, or

mediation will each reach a net change of zero when one out of the three takes over the entire population.

When the coefficients are all equal, this would represent an environment where none of the methods are particularly favored, making changes based on population interactions. For such an environment, the number of people belonging to the method holding the initial majority will approach the total population as time goes on. However, this will not always be the case, as the environment may have an aversion to or tend to favor one method over the others. As represented by different rate constants, when the environment shows some bias for or against a group, both the population size and the direction of the constants play into which group takes over the entire

To look at the addition of new groups, there are two possible cases. When new minority groups move into an existing population, they also tend to conform if there is an existing majority method. If a larger group is to join the population however, their method will become the dominant one as long as the existing environment is not too biased.

To take a look at the big picture, refugees in such diverse settlements would likely end up following the majority when settling disputes unless the initial environment already strongly favors a certain method or has an aversion to one. In terms of why this happens, we can adopt an example of where negotiation happens to be the method heavily favored by the environment. We can expect that as time passes, more and more of the population will join the negotiation group. This is out of habit and convenience and will continue until the other two methods die out due to a lack of use and a need to conform in order to mitigate conflicts. The main goal in all of these settlements is to maintain a peaceful coexistence even with the presence of diverse groups. Because of that, conformity to a majority or communally recognized standard is the most rational solution, and also serves to explain our results.

There are, of course, some limitations to this model, such as our assumption of all people being open to converting new methods, which is not entirely realistic in a real-world context.

References

[1] Vancluysen, Sarah and Ingelaere, Bert, *How refugees resolve disputes: insights from a Ugandan settlement.* The Conversation - (2020.07.28) Bronbestand: https://theconversation.com/how-refugees-resolve-disputes-insights-from-a-ugandan-settlement-142508 [c:irua:170563]

[2] *Figures at a Glance*, The UN Refugee Agency (2021). https://www.unhcr.org/en-us/figures-at-a-glance.html

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