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Consecutive Prime and Highly Total Prime Labeling in Graphs

By *Robert Scholle*

Abstract. This paper examines the graph-theoretical concepts of consecutive prime labeling and highly total prime labeling. These are variations on prime labeling, introduced by Tout, Dabboucy, and Howalla in 1982. Consecutive prime labeling is defined here for the first time. Consecutive prime labeling requires that the labels of vertices in a graph be relatively prime to the labels of all adjacent vertices as well as all incident edges. We show that all paths, cycles, stars, and complete graphs have a consecutive prime labeling and conjecture that all simple connected graphs have a consecutive prime labeling.

This paper also expands on work introduced by Ramasubramanian and Kala on total prime graphs in 2012 and Gnanajothi and Suganya on highly total prime graphs in 2016. We extend previous results by showing that no wheel graph or hypercube graph has a highly total prime labeling. We also show that no star graph with 8 or more vertices has a highly total prime labeling. In addition, we introduce millipede graphs, a new subfamily of caterpillar graphs, and show that certain millipede graphs do not have a highly total prime labeling.

1 Introduction

Graph theory is an area of discrete mathematics that deals with the study of graphs, which are structures that represent the pairwise relationships among discrete objects. A graph $G = (V, E)$ consists of a set of vertices, V, and a set of edges, E. Each edge connects two distinct elements of V. Vertices are generally represented pictorially as points, and edges are represented by line segments connecting these points. If we require that the edges are between distict vertices and and no two vertices have multiple edges between them, then the graphs are called simple.

Graph theory has obvious utility in its applications in the sciences. However, even when viewed apart from these applications, it yields beautiful mathematical gems, and

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can be used as a lens to study other areas of mathematics. In this paper, we use graph theory, and specifically graph labeling, as a lens to study prime numbers. Graph labeling is comprehensively surveyed in Gallian's 2016 paper [2]. Vertex labeling is a topic in graph theory introduced by Rosa in 1967 [6] in which values are assigned to the vertices of graphs according to various rules in order to understand the properties of the graphs. Edge labeling is similar but the edges are labeled instead of the vertices.

Prime graph labeling was introduced by Tout, Dabboucy, and Howalla in 1982 [8]. In prime graph labeling, vertices are labeled with distinct positive integers less than or equal to the number of vertices in the graph such that labels of adjacent vertices are relatively prime.

Deretsky, Lee, and Mitchem considered the labeling of edges rather than vertices and proposed vertex prime labeling in 1991 [1]. In vertex prime labeling, the edges are labeled with distinct positive integers less than or equal to the number of edges in the graph such that for each vertex of degree at least 2 the greatest common divisor of the labels of its incident edges is 1.

In this paper, we introduce a new variation on prime labeling and vertex prime labeling called consecutive prime labeling. In consecutive prime labeling, the vertices and edges are labeled with distinct positive integers less than or equal to the sum of the number of vertices and edges in the graph such that the label of every vertex is relatively prime to the label of each adjacent vertex and each incident edge. We show that several families of graphs have a consecutive prime labeling and conjecture that, in fact, all simple graphs have a consecutive prime labeling.

In 2012, Ramasubramanian and Kala combined the ideas of prime labeling and vertex prime labeling into total prime labeling [5]. In total prime labeling, the vertices and edges are labeled with distinct positive integers less than or equal to the sum of the number of vertices and edges in the graph such that two conditions are met: First, the labels of any two adjacent vertices are relatively prime. Second, the greatest common divisor of the labels of all edges incident to the same vertex is 1.

Gnanajothi and Suganya expanded on this work and introduced highly total prime labeling in 2016 [3]. Highly total prime labeling adds the restriction to total prime labeling that for each vertex of degree at least 2, any two edges that are incident to the same vertex have labels that are relatively prime. Note that highly total prime graphs are the same as total prime graphs except that instead of requiring simply that the labels of the edges incident to the same vertex have a greatest common divisor of 1, highly total prime graphs require the stronger condition that the edges be pairwise relatively prime. Thus all highly total prime graphs are total prime, but the converse is not true. We prove that several graphs are not highly total prime.

In Section 2, we summarize some useful properties of greatest common divisors which are used throughout the paper. In Section 3, we introduce and discuss consecutive prime labeling. In Section 4, we discuss highly total prime labeling.

Throughout this paper, we refer to vertices or edges being relatively prime to each other. When we say this, we imply that the associated vertex or edge labels are relatively prime. We adopt this modified terminology for the sake of conciseness.

2 Properties of Relatively Prime Numbers

We collect many well known, but important properties of Bezout's identity in the following proposition which will be useful throughout the article.

Definition 2.1. Two integers are **relatively prime** if their greatest common divisor is 1.

Bezout's Identity

The greatest common divisor of two nonzero integers a and b is 1 if and only if $ax + by = 1$ for some $x, y \in \mathbb{Z}$.

Proposition 2.1. *1. Two consecutive nonzero integers are relatively prime.*

2. The number 1 *is relatively prime to every natural number*

3. Consecutive odd integers are relatively prime.

4. Any prime number is relatively prime to any integer which it does not divide.

Proof. 1. We see that this is true by considering Bezout's identity with $a = n$, $b = n+1$. If $x = -1$ and $y = 1$, then the equation is satisfied.

2. We see that this is true by considering Bezout's identity with $a = 1$, $b \in \mathbb{N}$. If $x = 1$ and $y = 0$, then the equation is satisfied.

3. We see that this is true by considering Bezout's identity with $a = 2n+1$ and $b = 2n+3$, $n \in \mathbb{Z}$. If $x = -(n+2)$ and $y = n+1$, the equation is satisfied.

4. This is clear because any prime number p is relatively prime to $n \in \mathbb{Z} < p$ by definition, and the only integers greater than p with which p could have a common divisor are the multiples of p.

□

3 Consecutive Prime Labeling in Graphs

In this section, we define a consecutive prime labeling. We also prove that all paths, cycles, stars and complete graphs have a consecutive prime labeling.

Definition 3.1. A simple connected graph G with p vertices and q edges has a **consecutive prime labeling** if the vertices and edges of G can be labeled $1, 2, \dots, p+q$ such that the labels of vertices are relatively prime to the labels of all adjacent vertices as well as all incident edges.

Theorem 3.1. All paths have a consecutive prime labeling.

Proof. Let P_n be the path with vertex set $\mathrm{V} = \{v_1, v_2, \dots, v_{n+1}\}$ and edge set $\mathrm{E} = \{v_i v_{i+1} : 1 \le i \le n\}$. Since P_n has $n+1$ vertices and n edges, $p+q = 2n+1$.

Assign label $2i-1$ to vertex v_i for $1 \le i \le n+1$. Note that all vertex labels are odd and the difference between adjacent vertex labels is 2. Thus, the labels of adjacent vertices are relatively prime by Proposition 2.1 (3).

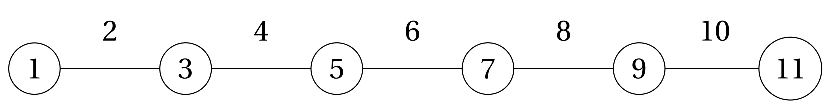
Assign label $2i$ to edge $v_i v_{i+1}$ for $1 \le i \le n$. The difference between the label of a vertex and the label of any incident edge is 1. Thus, the labels of vertices and their incident edges are relatively prime by Proposition 2.1 (1).

Since the vertices and edges are labeled 1 through $p+q$ and each vertex is relatively prime to all adjacent vertices and all incident edges, all paths have a consecutive prime labeling.

□

Figure 1 uses the prescribed labeling method on a path with 6 vertices.

Figure 1: Consecutive prime labeling of P_5



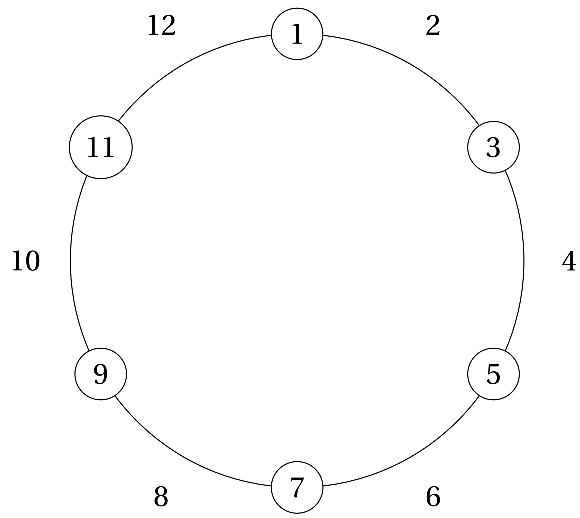
Theorem 3.2. All cycles have a consecutive prime labeling.

Proof. Let C_n be the cycle with vertex set $\mathrm{V} = \{v_1, v_2, \dots, v_n\}$ and edge set $\mathrm{E} = \{v_i v_{i+1} : 1 \le i \le n-1\} \cup \{v_1 v_n\}$. Since C_n has n vertices and n edges, $p+q = 2n$.

Assign label $2i-1$ to vertex v_i for $1 \le i \le n$. Since v_1 is given the label 1, it is relatively prime to v_n by Proposition 2.1 (2). For all other pairs of adjacent vertices, note that all vertex labels are odd and the difference between the labels of adjacent vertices is 2. So by Proposition 2.1 (3) the labels of all adjacent vertices are relatively prime.

Assign label $2i$ to edge $v_i v_{i+1}$ for $1 \le i \le n-1$, and assign label $2n$ to edge $v_1 v_n$. Except for the pair v_1 and $v_1 v_n$, the difference between the label of a vertex and the label of any incident edge is 1 so they are reletively prime by Proposition 2.1 (1). Since v_1 is given the label 1, it is relatively prime to edge $v_1 v_n$ by Proposition 2.1 (2). Thus, the labels of vertices and their incident edges are relatively prime.

Figure 2: Consecutive prime labeling of C_6



Since the vertices and edges are labeled 1 through $p+q$ and each vertex is relatively prime to all adjacent vertices and all incident edges, all cycles have a consecutive prime labeling.

□

Figure 2 uses the prescribed labeling method on a cycle with 6 vertices.

Theorem 3.3. All star graphs have a consecutive prime labeling.

Proof. Let $K_{1,n}$ be the star graph with vertex set $V = \{v_1, v_2, \dots, v_{n+1}\}$ and edge set $E = \{v_1 v_i : 2 \le i \le n+1\}$. Since $K_{1,n}$ has $n+1$ vertices and n edges, $p+q = 2n+1$.

Assign label $2i-1$ to vertex v_i for $1 \le i \le n+1$. Note that every edge in the graph has the vertex with label 1 as one of its endpoints. Thus, the labels of adjacent vertices are relatively prime by Proposition 2.1 (2).

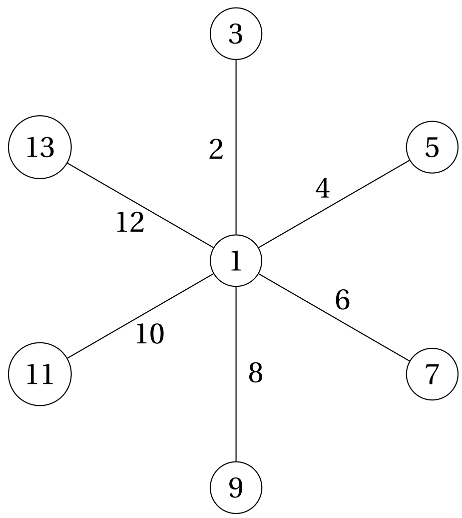
Assign label $2i$ to edge $v_1 v_{i+1}$ for $1 \le i \le n$. For all vertices except for the vertex with label 1, the difference between the label of the vertex and the label of its incident edge is 1. Thus, the labels of vertices and their incident edges are relatively prime by Proposition 2.1 (1).

Since the vertices and edges are labeled 1 through $p+q$ and each vertex is relatively prime to all adjacent vertices and all incident edges, all star graphs have a consecutive prime labeling.

□

Figure 3 uses the prescribed labeling method on a star with 7 vertices.

Figure 3: Consecutive prime labeling of $K_{1,6}$



Theorem 3.4. All complete graphs have a consecutive prime labeling.

Proof. Let K_n be the complete graph with $n = p$ vertices. By Proposition 2.1 (4), any prime number greater than $\frac{p+q}{2}$ must be relatively prime to all other integers less than or equal to $p+q$, since it does not divide any of these integers. So if every vertex could be labeled with 1 or a prime number greater than $\frac{p+q}{2}$, then the graph would be consecutive prime.

Let $\pi(x)$ be the number of primes less than or equal to x. When $x \geq 55$, lower and upper bounds on $\pi(x)$ are given by $\frac{x}{\ln(x+2)} < \pi(x) < \frac{x}{\ln(x-4)}$ (see Rosser [7]).

Let r be the number of primes less than or equal to x but greater than $\frac{x}{2}$. It follows from above that, for $x \geq 110$,

$$r \geq \left\lceil \frac{x}{\ln(x+2)} \right\rceil - \left\lfloor \frac{x}{2\ln(\frac{x}{2}-4)} \right\rfloor .$$

If a complete graph has p vertices and q edges, then $q = \frac{p(p-1)}{2}$. (This is because each of the p vertices is adjacent to $p-1$ other vertices, but then we have counted every edge twice, so we must divide by two.) So we have the following:

$$p^2 - p = 2q$$
$$4p^2 - 4p + 1 = 8q + 1$$
$$4p^2 + 4p + 1 = 8p + 8q + 1$$
$$(2p+1)^2 = 8p + 8q + 1$$
$$p = \frac{-1 + \sqrt{8(p+q)+1}}{2}.$$

Let $x = p + q$. All of the vertices of the complete graph can be labeled with 1 or a prime number greater than $\frac{p+q}{2}$ if $r + 1 \geq p$. It is the case that

$$r + 1 \geq \left\lceil \frac{x}{\ln(x+2)} \right\rceil - \left\lfloor \frac{x}{2\ln(\frac{x}{2} - 4)} \right\rfloor + 1 \geq \frac{-1 + \sqrt{8x+1}}{2} = p$$

for all integer values of $p \geq 23$.

Let K_n be the complete graph with n vertices. So K_n has a consecutive prime labeling for $n \geq 23$.

Now consider graphs of order $19 \leq p \leq 22$ and label the vertices of each of these graphs the following way. Label one vertex 1, and the rest of the vertices with the first $p - 1$ prime labels greater than $\frac{p+q}{2}$. It is easily verifiable that one more than the number of prime labels larger than $\frac{p+q}{2}$ is greater than or equal to the number of vertices in each of these graphs. This is shown explicitly in Table 1.

Table 1: Number of Primes to Label Vertices

Number of vertices, p	Number of prime labels greater than $\frac{p+q}{2}$
19	18
20	19
21	20
22	24

If every vertex is labeled as stated above, then any edge can receive any other label. Thus, K_n has a consecutive prime labeling for $n \geq 19$.

Now consider the cases $4 \leq p \leq 18$. Label the vertices as shown in Table 2.

The method of labeling shown in Table 2 uses 1 and the highest prime numbers less than $p + q$ to label the vertices. In this way we minimize the number of edge labels that are multiples of a vertex label; there are very few such labels. In Table 3, we provide assignments for each of these remaining edge labels such that the corresponding edges are relatively prime to their incident vertices.

Table 2: Labels of Vertices

Number of Vertices	Labels of Vertices
4	7, 5, 3, 1
5	13, 11, 7, 5, 1
6	19, 17, 13, 11, 7, 1
7	23, 19, 17, 13, 11, 7, 1
8	31, 29, 23, 19, 17, 13, 11, 1
9	43, 41, 37, 31, 29, 23, 19, 17, 1
10	53, 47, 43, 41, 37, 31, 29, 23, 19, 1
11	61, 59, 53, 47, 43, 41, 37, 31, 29, 23, 1
12	73, 71, 67, 61, 59, 53, 47, 43, 41, 37, 31, 1
13	89, 83, 79, 73, 71, 67, 61, 59, 53, 47, 43, 41, 1
14	103, 101, 97, 89, 83, 79, 73, 71, 67, 61, 59, 53, 47, 1
15	113, 109, 107, 103, 101, 97, 89, 83, 79, 73, 71, 67, 61, 59, 1
16	131, 127, 113, 109, 107, 103, 101, 97, 89, 83, 79, 73, 71, 67, 61, 1
17	151, 149, 139, 137, 131, 127, 113, 109, 107, 103, 101, 97, 89, 83, 79, 73, 1
18	167, 163, 157, 151, 149, 139, 137, 131, 127, 113, 109, 107, 103, 101, 97, 89, 83, 1

Figure 4 uses the labeling given by Tables 2 and 3 on a complete graph with 6 vertices. The labels in bold cannot be placed on an edge that is incident to the vertex with label 7, but they can be used to label any other edge. The labels marked with an asterisk can be placed on any edge without conflict.

Thus, K_n has a consecutive prime labeling for $4 \leq n \leq 18$.

The case $p = 3$ is an example of a cycle and $p = 2$ is an example of a path. All paths and cycles have already been proven to be consecutive prime in Theorems 3.1 and 3.2, respectively.

Therefore, every complete graph has a consecutive prime labeling.

□

We conjecture that all simple connected graphs have a consecutive prime labeling. This is because the most difficult graphs to label should be complete graphs, since every vertex is connected to every other and complete graphs have the greatest number of edges. However, it turns out that there are enough prime numbers to label complete graphs with ease, so it seems that all simple connected graphs should have a consecutive prime labeling.

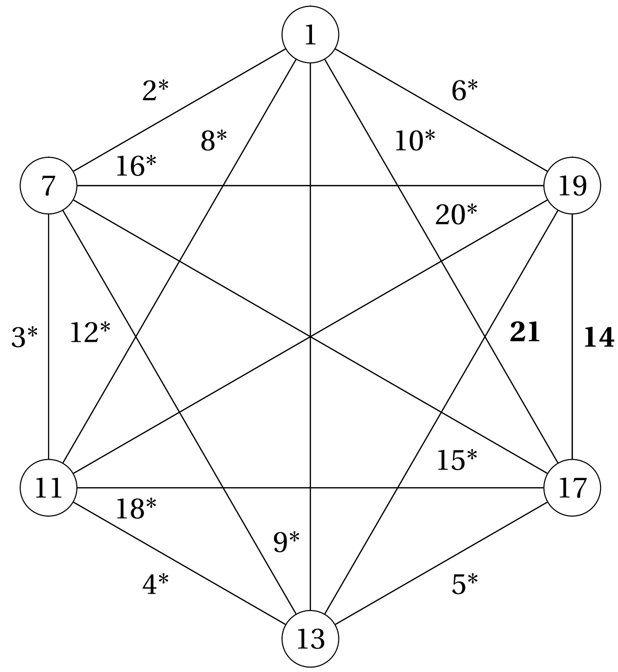
Table 3: Labels of Edges

Number of vertices	Edge between	Label of edge
4	7-5	6
	7-3	10
	7-1	9
5	13-11	14
	13-7	10
	13-1	15
6	19-17	14
	19-13	21
7	23-19	26
	23-17	22
	23-13	14
	23-11	21
	23-1	28
8	31-29	34
	31-23	26
	31-19	22
	31-17	33
9	43-41	38
	43-37	34
10	53-47	46
	53-43	38
11	61-59	62
	61-53	58
	61-47	46
12	73-71	74
	73-67	62
13	89-83	86
	89-79	82
14	101-103	94
15	113-109	118
16	131-127	134
	131-113	122
17	151-149	146
18	167-163	166

4 Highly Total Prime Labeling in Graphs

Definition 4.1. A simple connected graph G with p vertices and q edges has a **highly total prime labeling** if the vertices of G can be labeled $1, 2, \ldots, p+q$ such that the label

Figure 4: K_6 with the labeling given by Tables 2 and 3



of any vertex v is relatively prime to the labels of all vertices to which v is adjacent, and labels of incident edges to v are pairwise relatively prime.

Paths, cycles of even length and graphs consisting of two cycles with a common edge have already been shown to be highly total prime by Gnanajothi and Suganya in 2016 [3]. In this section, we consider wheels, hypercube graphs, stars and millipede graphs.

Figure 5 shows a star of 7 vertices with a highly total prime labeling. As we will show in Theorem 4.3, this is the largest star graph that has a highly total prime labeling.

Theorem 4.1. No wheel graph has a highly total prime labeling.

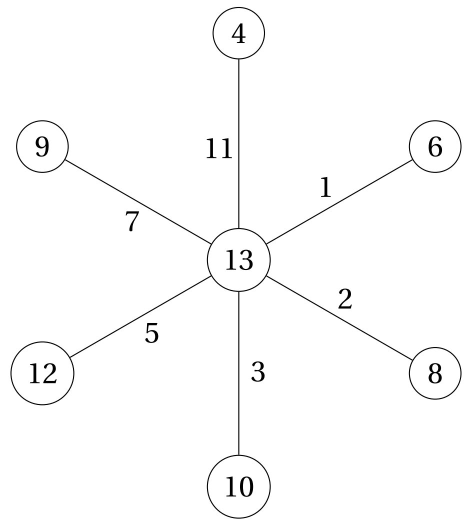
Proof. Let W_n be the wheel with vertex set $V = \{v_1, v_2, \ldots, v_{n+1}\}$ and edge set $E = \{v_1 v_i : 2 \leq i \leq n+1\} \cup \{v_i v_{i+1} : 2 \leq i \leq n\} \cup \{v_2 v_{n+1}\}$. This means that W_n has $2n$ edges and $n+1$ vertices. The total number of labels in such a graph is $3n+1$. Edges can only have labels that share a common factor if they share no vertex.

The size of a maximum independent set, or largest set of pairwise non-adjacent vertices, is given by

$$p_1 = \left\lfloor \frac{n}{2} \right\rfloor.$$

The size of a maximum independent edge set, or largest set of pairwise non-adjacent

Figure 5: Highly total prime labeling of $K_{1,6}$



edges, is given by

$$q_1 = \left\lfloor \frac{n+1}{2} \right\rfloor.$$

So $p_1 + q_1 = n$ is the maximum number of vertices and edges that can receive an even label.

And the number of even labels in a wheel graph is given by

$$e = \left\lfloor \frac{3n+1}{2} \right\rfloor.$$

And $p_1 + q_1 = n < e$ for all n. So, for any wheel graph, there are not enough labels for all of the even labels to be placed without conflict, so no wheel graph is highly total prime.

□

Theorem 4.2. No hypercube graph of dimension 3 or greater has a highly total prime labeling.

Proof. A hypercube graph with 2^n vertices is the Cartesian product of n paths of length 2. Consider a hypercube graph of dimension n, the graph consisting of the vertices and edges of an n-dimensional hypercube. Such a graph has 2^n vertices and $2^{n-1}n$ edges. Edges can only have labels that share a common factor if they share no vertex.

The size of a maximum independent set in a hypercube graph, p_1, is given by (see Harary [4])

$$p_1 = 2^{n-1}.$$

The size of a maximum independent edge set in a hypercube graph, q_1, is given by (see Harary [4])

$$q_1 = 2^{n-1}.$$

So $p_1 + q_1 = 2^n$ is the maximum number of vertices and edges that can receive an even label.

And the number of even labels in a hypercube graph is given by

$$e = 2^{n-2}(n+2).$$

And $p_1 + q_1 = 2^n < e$ for all $n > 2$. So for any hypercube graph other than a square there are not enough labels for all of the even labels to be placed without conflict, so no hypercube graph of dimension 3 or greater is highly total prime.

□

Theorem 4.3. No $K_{1,n}$ graph where $n \geq 7$ has a highly total prime labeling.

Proof. Let $K_{1,n}$ be the star with $n+1$ vertices and n edges. The total number of labels in such a graph is $2n+1$.

When $n < 7$ it is not difficult to show that $K_{1,n}$ is highly total prime.

Now consider the $K_{1,n}$ graph where $n \geq 7$. The number of even labels that must be assigned is n, and no more than one edge can be assigned an even label.

Suppose that no edge were to be assigned an even label; then all of the even labels would be used to label all n of the outer vertices. Since at most one edge can be assigned a label that is an odd multiple of 3, but there are at least three such labels for $n \geq 7$, this labeling is not possible.

Now suppose, instead, that exactly one edge were to be assigned an even label; then the remaining $n-1$ even labels would be used to label all but one of the outer vertices. Again, at most one edge can be assigned a label that is an odd multiple of 3. The two remaining vertices are adjacent and thus cannot both be assigned an odd multiple of 3 label.

Therefore, no $K_{1,n}$ graph where $n \geq 7$ has a highly total prime labeling. □

Figure 5 is a $K_{1,6}$ graph. It is the largest star graph that has a highly total prime labeling. If we consider a star with one more vertex, then we have 10 labels which are even or multiples of 3. This means that there is no way for the graph to be labeled without two edges or two vertices being relatively prime.

Definition 4.2. Define an n, a-**millipede graph** to be a graph comprised of a main path of a vertices and $n-a$ "legs," where a leg consists of a vertex of degree 1 and a single edge connecting that vertex to the path, under the condition that the difference in degree between any two path vertices, or "centers," is 1. Note that n is therefore the total number of vertices in the graph. Note also that the millipede graphs are a subfamily of caterpillar graphs, which are graphs in which every vertex is on a central stalk or only one edge away from that central stalk.

Figure 6 is a 14,4-millipede graph with a highly total prime labeling.

Theorem 4.4. No n, a-millipede graph where each center has degree at least 4 and $n > 6(a+1)$ has a highly total prime labeling.

Proof. Edges can only have labels that share a common factor if they share no vertex. So each center vertex can have at most one incident edge with an even label and at most one incident edge with a label that is an odd multiple of 3. The only remaining potential candidates to be assigned even labels or labels that are odd multiples of 3 are the n vertices.

This implies that no more than $2a + n$ edges and vertices can be assigned even labels or labels that are odd multiples of 3.

The number of even labels in a millipede graph is

$$e = n - 1.$$

And the number of labels in a millipede graph that are odd multiples of 3 is

$$o = \left\lfloor \frac{n+1}{3} \right\rfloor \geq \frac{n}{3} - 1.$$

When the number of even labels and labels that are odd multiples of 3 exceeds the number of edges and vertices that can be assigned these labels then the graph is not highly total prime.

So when $2a + n < e + o$ the graph is not highly total prime.

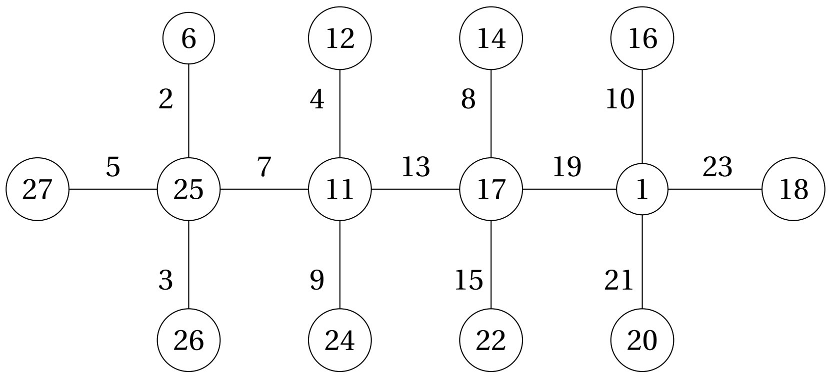
So no millipede graph where $2a + n < (n-1) + (\frac{n}{3} - 1)$ is highly total prime. This implies that no millipede graph where $n > 6(a+1)$ is highly total prime.

□

We conjecture that Theorem 4.4 could actually have the stricter restriction that $n > 3(a+2)$. This inequality is obtained by assuming that the even numbers and odd multiples of 3 cannot be used to label the center vertices, thus implying that no more than $2a + (n-a) = a + n$ edges and vertices can be assigned even labels or labels that are odd multiples of 3. If one were to label a center with an even number, none of the vertices adjacent to that center could be assigned an even label, but if the center were to be assigned a prime label, then every vertex adjacent to that center could be assigned an even label. However, we have not yet been able to prove that we must exclude the centers from consideration when placing the even labels and labels that are odd multiples of 3, so we cannot prove this tighter restriction.

Note that when using highly total prime labeling, a graph with one vertex with very high degree is very difficult to label. If a number is used to label an edge of such a vertex,

Figure 6: Millipede graph with highly total prime labeling



then no other multiple of that number may be used on a co-incident edge. However, if many of the vertices in the graph have the same degree, such as in a cycle or path, then highly total prime labeling becomes much easier.

5 Conclusion

In this paper, we have introduced consecutive prime labeling and proved that several families of graphs have a consecutive prime labeling. We have also expanded on previous work by Gnanajothi and Suganya [3] on highly total prime graphs by proving that several families of graphs do not have a highly total prime labeling. We conjecture that all simple graphs have a consecutive prime labeling. We also conjecture that the theorem involving the millipede graph could have the stricter restriction that $n > 3(a+2)$.

Prime graphs give us an opportunity to study prime numbers from a different perspective. The main difficulty in this type of research is that the prime numbers are sporadically spaced, which makes them difficult to work with, especially when they are considered in the context of graphs. However, graph structures allow us to see the relationships among prime numbers in a unique way.

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