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Detecting Forged Handwriting with Wavelets and Statistics

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1 Introduction: Art & Forgeries

For centuries, the forgeries of great artists have held a place of fascination in the public eye. Museums occasionally contain entire galleries dedicated to those forgeries which are considered to be "museum quality." People such as Hans van Meegeren and Elmyr de Hory have created such masterful imitations of the works of Johannes Vermeer, Henri Matisse, and Pablo Picasso that they have become famous for their ability to deceive art critics and historians [5, 8]. As these imitators have been discovered by different methods of analysis, the following questions arise:

- What exactly distinguishes an authentic work of art from a forgery?
- Does this question have a mathematical answer?
- How are these questions related to the problem of determining if a handwriting sample is a forgery?

Art historians and experts use a combination of subjective and objective techniques to determine the authenticity of a work in question. Style, content, and chemical composition of a painting are very important in the classification of a work of art. An artist's brush stroke, portrayal of light, and use of color can all be considered part of his or her stylistic signature. Content is also quite important; some forgeries have been discovered through anachronisms contained within the painting. The chemical composition of the paints used and the actual material of the canvas or paper can also be instrumental in dating a piece of art, but only lately have these objective methods of chemical analysis become more widely used.

Recently, Lyu, Rockmore and Farid of Dartmouth College described a new, mathematical method for determining the authenticity of works of art [9, 10, 11]. Their method applies a wavelet-like transform and basic statistics to greyscale versions of paintings and sketches. The idea behind the method of Lyu *et al.* is to differentiate between authentic works and imitations by analyzing the consistency of the data received from the wavelet-like decomposition of the images. In theory, though a forger tries to imitate another artist's style, his or her work should be statistically different from that of the true artist.

Lyu *et al.* used their method to first discriminate between authentic works and imitations in a group of thirteen different sketches, each of which had at one time been attributed to Pieter Bruegel the Elder. They also applied their method to the painting "Madonna with Child" by Pietro di Cristoforo Vannucci (Perugino) and analyzed the faces of the subjects in the painting to determine whether or not the faces had all been painted by the same artist. The mathematical analysis suggested that some of the Bruegel sketches were done by imitators, while some of the Perugino faces were perhaps done by Perugino's apprentices. These results were supported by the existing opinions of art historians. The results also seemed to indicate that an artist could have some kind of "mathematical signature," with the data from one artist falling within a certain statistical range and the data of imitators falling outside of that range.

While there is criticism of their work (e.g. [11] and below), their basic idea appears sound, and a logical extension is to apply the method to the analysis of handwriting. The problem is to determine if there is a mathematical "signature" that corresponds to a person's handwriting. In this paper, we discuss how we modified the method of Lyu *et al.*, by using a two-dimensional wavelet filter, a larger number of samples, and a simpler statistical analysis, in order to determine the difference between authentic and forged handwriting.

2 Handwriting Analysis

Handwriting forgery has a history as long as or perhaps longer than that of art forgery, and average people are more likely to encounter handwriting forgery in the form of identity theft or signature forgery. In addition to these more familiar cases of handwriting forgery, there have been more exceptional instances in which people have tried to make money by forging. Perhaps the most extravagant and infamous case occurred in 1972, when a man by the name of Clifford Irving claimed to have collaborated with Howard Hughes on an autobiography for the billionaire recluse. He forged letters from Hughes to a publishing company, encouraging the company to buy the manuscript. Irving received a $765,000 advance from the publishing company before Hughes came out of his seclusion and denounced Irving as a fraud [7].

Handwriting samples are frequently used as forensic evidence (ransom notes, letters, etc.), and the identification of the writer is often crucial to the case. Handwriting analysts look at four things when they compare samples of handwriting to determine authenticity: form, line quality, arrangement, and content [12]. Form involves the general shape of the letters, their proportionate size, and any slanting, curving, connections, or retracing. The line quality includes the type of writing instrument used, the amount of pressure used by the writer, and the continuity of the script itself. The arrangement of the writing refers to the spacing and alignment of the characters, and content is any unique or identifying spelling, grammar, or punctuation. In fact, the distinguishing characteristics of a writing sample are somewhat similar to those of a piece of art. In the case of a forgery of a historical document, the type of ink and paper used would also be taken into consideration.

The algorithm described in this paper describes how three of these four aspects of handwriting (all but content) can be aggregated into a set of numbers by first applying a wavelet transform to create wavelet coefficients, then creating what is called a *linear predictor* for the wavelet coefficients, and finally applying descriptive statistics.

3 Wavelet Filters

Wavelets have become an important tool in applied mathematics over the past 15 years. As an alternative to Fourier analysis, they allow researchers to study images and other signals at different resolutions. One introductory text on wavelets is [3].

Our algorithm makes use of bivariate filters, which operate on $2^n \times 2^n$ matrices. These filters transform one such matrix into another, with the idea being that the entries in the new matrix may be a rearrangement of information that will help with the application at hand, in this case, handwriting analysis.

The simplest bivariate filters are the four Haar filters. Labelled H, G_v, G_h, and G_d, they are called the *low-pass*[1], *vertical high-pass*, *horizontal high-pass*, and *diagonal high-pass* filters, respectively [2, 4]. The four filters are represented by these matrices:

$$H = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{bmatrix} \qquad G_v = \begin{bmatrix} \frac{1}{2} & -\frac{1}{2} \\ \frac{1}{2} & -\frac{1}{2} \end{bmatrix}$$

$$G_h = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ -\frac{1}{2} & -\frac{1}{2} \end{bmatrix} \qquad G_d = \begin{bmatrix} \frac{1}{2} & -\frac{1}{2} \\ -\frac{1}{2} & \frac{1}{2} \end{bmatrix}.$$

To filter the matrix

$$Q = \begin{bmatrix} w & x \\ y & z \end{bmatrix},$$

each entry is multiplied by the corresponding entry in the filter, and the results are added together. For example, applying the diagonal high-pass filter yields

$$g_d = G_d[Q] = \frac{1}{2}w - \frac{1}{2}x - \frac{1}{2}y + \frac{1}{2}z.$$

[1]It would seem that the high-pass filters should be labeled with H's, but that is not the convention.

A concrete way to think of filtering is that the low-pass filter computes an average of matrix entries, while the high-pass filters measure changes between adjacent rows, columns, and diagonals.

After all four filters have been applied, the results are put into in a new matrix,

$$\hat{Q} = \begin{bmatrix} h & g_v \\ g_h & g_d \end{bmatrix}.$$

In the case of $n = 2$, filtering is an iterative, two-step process. We begin by partitioning a 4×4 matrix into four 2×2 blocks,

$$Q = \left[\begin{array}{c|c} Q_{nw} & Q_{ne} \\ \hline Q_{sw} & Q_{se} \end{array}\right].$$

When the filter H is applied to each of the sub-matrices, the result is the 2×2 matrix

$$H[Q] = \begin{bmatrix} H[Q_{nw}] & H[Q_{ne}] \\ H[Q_{sw}] & H[Q_{se}] \end{bmatrix}.$$

Next, we apply the other three filters to the four sub-matrices of Q, creating three more 2×2 matrices $G_v[Q]$, $G_h[Q]$, and $G_d[Q]$, and place the results in a new 4×4 matrix in a natural way:

$$Q_1 = \begin{bmatrix} H[Q] & G_v[Q] \\ G_h[Q] & G_d[Q] \end{bmatrix}.$$

The second step is to use the 2×2 matrix $H[Q]$ as input into another round of filtering, in the same manner as described earlier for $n = 1$. The result is a new 2×2 matrix

$$Q_2 = \begin{bmatrix} H[H[Q]] & G_v[H[Q]] \\ G_h[H[Q]] & G_d[H[Q]] \end{bmatrix},$$

which replaces $H[Q]$ in Q_1 above. We call the final matrix $\hat{Q}$, where

$$\hat{Q} = \begin{bmatrix} Q_2 & G_v[Q] \\ G_h[Q] & G_d[Q] \end{bmatrix}.$$

The entries in $\hat{Q}$ are called *wavelet coefficients.* Here is an example of Q and $\hat{Q}$:

$$Q = \left[\begin{array}{cc|cc} 1 & 6 & -2 & 3 \\ -5 & 0 & 5 & 8 \\ \hline 2 & 7 & 9 & -4 \\ -3 & 10 & -1 & 4 \end{array}\right] \qquad \hat{Q} = \left[\begin{array}{cc|cc} 10 & -1 & -5 & -4 \\ -2 & 5 & -9 & 4 \\ \hline 6 & -6 & 0 & -1 \\ 1 & 1 & 4 & 9 \end{array}\right].$$

In this example, we can refer to entries in $\hat{Q}$ as *children*. The child 0 then has *neighbors* -1, 4 and 9, *vertical and horizontal cousins* -5 and 6, and *parents* 1, 6, -5, and 0 in Q. These terms are important below when discussing linear predictors.

For our algorithm, we use 4×4 filters that are two-dimensional analogues of the Daubechies wavelet family [1]. Using these filters requires some slight padding of the edges of the image regions we analyze (see below). Each region is passed under each of the four filters. The low pass filter result is then padded and filtered again. Here is an example of one of these filters, namely the low pass filter:

$$H = \begin{bmatrix} -.25 + .125\sqrt{-46+28\sqrt{3}} & .125\sqrt{-46+28\sqrt{3}} & .5 - .25\sqrt{3} - .125\sqrt{-46+28\sqrt{3}} & .25 - .25\sqrt{3} + .125\sqrt{-46+28\sqrt{3}} \\ 1 - .5\sqrt{3} & 1 - .5\sqrt{3} & -.25 + .25\sqrt{3} & -.25 + .25\sqrt{3} \\ .5 - .25\sqrt{3} - .125\sqrt{-46+28\sqrt{3}} & .75 - .25\sqrt{3} + .125\sqrt{-46+28\sqrt{3}} & .25 + .5\sqrt{3} + .125\sqrt{-46+28\sqrt{3}} & .5\sqrt{3} - .125\sqrt{-46+28\sqrt{3}} \\ -.75 + .25\sqrt{3} & -.25 + .25\sqrt{3} & 1 & 0.5 \end{bmatrix}$$

The other filters are created by multiplying selected entries of H by -1, in a manner analagous to the Haar bivariate filters.

4 Linear Predictors

It has been observed that in an image, nearby wavelet coefficients tend to be similar (since they are being created from similar pixel values) [6]. This idea leads to the concept of a linear predictor, which is a formula to approximately determine a coefficient as a function of its neighbors. A simple example is to take a matrix of values and write each entry in the matrix as a linear combination of its neighbors. For example, if v is a value with neighbors l, r, u and d, then the question becomes what values of w_1 through w_4 will satisfy

$$v = w_1 l + w_2 r + w_3 u + w_4 d$$

.

For each entry in the matrix an equation similar to the above is written, using the same unknowns w_1 through w_4. Thus we get a system of linear equations with four unknowns, which can be written more generally as $\vec{V} = Q\vec{w}$.

Because this system of equations is usually inconsistent, we determine a least squares solution $\vec{w}$ by minimizing the quadratic error function $E(\vec{w}) = [\vec{V} - Q\vec{w}]^2$.

Since

$$Q\vec{w} = \begin{bmatrix} \sum_{k=1}^{n} q_{1,k}w_k \\ \sum_{k=1}^{n} q_{2,k}w_k \\ \vdots \\ \sum_{k=1}^{n} q_{n,k}w_k \end{bmatrix}$$

where $q_{i,k}$ is the entry in the ith row and the kth column of the matrix Q, the Jacobian of $Q\vec{w}$ is simply the transpose of Q:

$$J(Q\vec{w}) = \begin{bmatrix} q_{1,1} & q_{2,1} & \cdots & q_{n,1} \\ \vdots & & & \vdots \\ q_{1,n} & q_{2,n} & \cdots & q_{n,n} \end{bmatrix} = Q^T.$$

Therefore,

$$\frac{dE(\vec{w})}{d\vec{w}} = 2Q^T[\vec{V} - Q\vec{w}]$$

and when solving $\frac{dE(\vec{w})}{d\vec{w}} = 0$, we have

$$\vec{w} = (Q^TQ)^{-1}Q^T\vec{V}.$$

Thus, $\vec{w}$ contains the best weighting values for the linear predictor and can be computed from Q and $\vec{V}$.

Applying this idea to wavelet coefficients, a system of equations are created by considering linear combinations of neighbors, cousins and parents of a child coefficient. Although there are 80 possible "relatives" for each child (due to applying the filters several times), the linear predictor we created to analyze handwriting makes use of the neighboring coefficients specified in the work of Buccigrossi and Simoncelli [6]. For the resulting coefficients from the vertical filter, the neighbors include the coefficients to the left and above the child, its parent, and its diagonal cousin. Similar neighbors are used in the analysis of the coefficients from the horizontal filter. However, five neighbors are used for the coefficients of the diagonal filter. These include the up and left neighbors, the parent, and cousins from the horizontal and vertical filters. If there is no up or left neighbor of the coefficient, zero is used instead.

After $\vec{w}$ is determined, the error involved in the linear predictor is calculated by

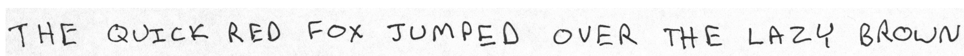
$$\log_2(\vec{V}) - \log_2(|Q\vec{w}|).$$

The vector $Q\vec{w}$ may contain negative entries so its absolute value must is used. Both $\vec{V}$ or $Q\vec{w}$ can have entries of zero, so those values are reset to 1×10^{-8} when calculating the error, which is a conventional adjustment to make.

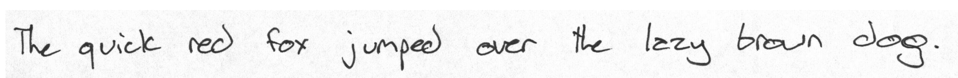
5 Algorithm Applied to Handwriting Samples

To begin our analysis of handwriting, we collected 22 samples of handwriting from 11 people and created four forgeries. Each person wrote the sentence "The quick red fox jumped over the lazy brown dog," in his or her standard handwriting, and again in capital letters on a sheet of white paper with a black pen. Four other samples were created by one of the authors to imitate four of the 22 samples. These images were scanned at 250 dpi into a .jpg format. Here are a few examples:

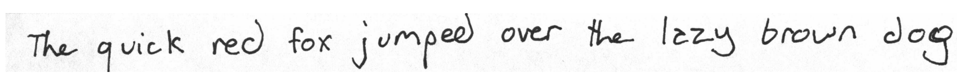
Beverly's Capitals



Beverly's Print



Caroline's Forgery of Beverly's Print



We used a program called Pic2Pic [13] to convert the .jpg files into portable greymap .pgm files. A .pgm file assigns to each pixel a value ranging from 0 (black) to 255 (white). Using a *Maple* worksheet, we converted each image from greyscale to black and white and then divided it into 256×256 pixel regions. (Dividing the image under analysis into smaller squares is a key part of the Lyu et al. method. Because of the nature of the wavelet filters, it was important that the side length of the squares were a power of 2.) We then processed each region using another *Maple* worksheet which used the 4×4 wavelet filter described earlier.

Next, we determined the weights and respective errors for the linear predictors. This led to six sets of data for each 256×256 pixel region. The six sets were composed of the horizontal weights, the vertical weights, the diagonal weights, and their corresponding errors.

In the method that we developed, statistics were very important in comparing the writing samples. In order to analyze the data sets, we calculated several central moments but focused on the third central moment, or *skewness*, which is a measure of how the data in a normal distribution is skewed either to the right or the left of the mean. In our algorithm, for each set of data from each region, we computed the skewness, leading to six other sets of data (horizontal skewness, vertical skewness, etc.).

To compare two handwriting samples, we applied a one-way ANOVA (analysis of variance) test on each of the six sets of skewness values. Using an ANOVA test, which determines the variance between and within different groups of data, helped determine whether the groups were statistically different or statistically similar. In *Maple*, a one-way ANOVA test yielded a number which was the standard (Student's) t-test result subtracted from a value of one. If all six of these values were below 0.95, we concluded that the compared handwriting samples were written by two different people. If one or more of the values was above 0.95, then the handwriting samples came from the same person.

For the handwriting samples we collected, our method determined in our earlier example that Caroline's forgery was distinct from Beverly's samples. It also concluded that both of Beverly's samples came from the same person. Our 26 samples led to 325 comparisons, and the algorithm determined correctly for 86% of the comparisons whether or not the samples were from the same person.

6 Further Comments

As stated earlier, we based our methods on the ones used by Lyu, Rockmore, and Farid. Their methods were applied to a group of thirteen landscape sketches, each of which had at one time been attributed to Pieter Brugel the Elder and to the regions containing the faces of those depicted in the painting "Madonna with Child" by Pietro di Cristoforo Vannucci (Perugino). After scanning and converting the images to greyscale, they were subdivided and were passed under five successive rounds of wavelet-like filtering. The result for each work of art was a point cloud in 72-dimensional space consisting of coefficient and error statistics for each region. At this step our methods diverge. Using Hausdorff distance, "authentication was indicated by the distance between these point clouds, with the belief that works by the same artist would be close together, irrespective of content, and that an imitation would be relatively far from the authenticated [works]." [9]

For landscape sketches, multidimensional scaling was applied to the 13×13 matrix containing the Hausdorff distances between each point cloud. Then, the center of mass was computed for the 13 points in 3-space. For this sample set, one-way ANOVA determined that the difference of the means of the distances of the points from the center of mass for the eight works previously authenticated by the Metropolitan Museum of Art versus the debunked works was statistically significant. However, Lyu *et al.* did not analyze other groupings of the sketches to determine if their distances also satisfied the vague requirement of being "relatively far" apart.

A similar process was used to analyze the faces within the Perugino's "Madonna with Child." However, even in collecting the data, we believe that Lyu *et al.* have ignored two key components of art authentication. First, only the six regions containing faces are used in the comparison. This sample size is too small to generate a mathematical signature for an artist. Second, while converting a charcoal sketch to greyscale maintains the integrity of the image, a significant amount of data is lost when performing this step on a painting. Overall we believe that while their method does extract essential data from sketches and drawings, there are questions about the method of data analysis used by Lyu *et al.* To be an objective tool in art authentication, this technique needs to be refined.

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