

## Improving Conservation Techniques Through Graph Theoretic Models

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**IMPROVING CONSERVATION  
TECHNIQUES THROUGH GRAPH  
THEORETIC MODELS**

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# IMPROVING CONSERVATION TECHNIQUES THROUGH GRAPH THEORETIC MODELS

Rob Hammond

**Abstract.** While concerns for local and global ecological issues increase, there is a growing need for the conservation of functioning ecosystems; as land management evolves in scope and emphasis, land managers are in need of new tools. Graph theory, a relatively efficient mathematical modeling approach, has been used for modeling and analyzing an array of networks and as a result proving itself as a potential framework for landscape modeling with its adaptable measures. Through an in-depth review of two unique examples of graph theoretic habitat modeling, we will see how these models compare to the more complex and biologically accurate spatially explicit population models, and what they can tell us about habitats that have had no previous analysis. It will then be seen that graph theoretic modeling approaches are important to habitat conservation research in various ways.

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## 1 Introduction

Natural resource managers today are encountering new challenges as land management is evolving in emphasis and scope. In the past wildlife conservation was based solely on maintaining resources, i.e., timber, game, and recreation, where individual sections of land were protected for one particular commodity with little concern for the interactions between habitats. Specifically, topics such as landscape connectivity and its affects on the long-term health of a species, and the advantages and disadvantages of the spatial location of a reserve site, have often been ignored in wildlife conservation. Now, due to increasing concerns for global ecological issues, there is a need for the conservation of functioning ecosystems. Public land managers must deal with entire landscapes and understand how events in one area will affect its surrounding regions. Consequently a need for new tools, particularly spatial models that allow for thorough understandings of the metapopulation structures within landscapes, has arisen [1].

Graph theory, a relatively efficient mathematical modeling approach, has been used to model and analyze networks ranging from epidemics to nanotechnology, creating connectivity measures with varying degrees of complexity from assumptions to algorithms. Therefore graph theory has proven to be a potential framework for spatial modeling of landscapes because of its existing and adaptable measures that can analyze at both a local and global level. The devised methods provide insight into the connectivity of landscapes, which is an attribute that is often overlooked in determining where reserve sites are created. Two of the many graph theoretic applications to ecology and landscape modeling are discussed in reference to this expository paper. These applications were chosen, as they are significantly different in their methods of population modeling and due to the robustness and approachability of their work.

In the first example, the research done on the connectivity at the individual reef level within a network, and graph theory's ability to create better understandings of the network functionality of the Cairns Section of the Great Barrier Reef (GBR) from Stuart J. Kinninmonth et al. [2] will be discussed. In a similar approach the methods of measuring connectivity of the habitat patches of the Wood Thrush population, a group of songbird that has been in decline over past decades, in Wake County, North Carolina will be examined. This example compares metrics used in the minimalistic graph theoretic models to the metrics used in the more biologically accurate spatially explicit population models (SEPMs), which is provided through a number of scholarly articles on the subject matter by Emily S. Minor and Dean L. Urban [3,5,7]. The goal of Minor and Urban's work was to determine if the graph theoretic models might be used to obtain a more efficient analysis of a habitat so that their work could act as an example for others utilizing these methods for any simulation models or species [3].

SEPMs are favored amongst ecologists as they are the most accurate method of population modeling due to their specific spatial arrangements and biologically oriented inputs. SEPMs allow ecologists to study species at the local, landscape, and global scales as well as observe possible responses to random and intentional disturbances in the environment, whether climate change, forest fires, oil spills, or human intervention [4].

Ecologists find that SEPMs can be impractical at times as they rely almost entirely on past research of a focal species due to the fact that the natural history of any species is not available, forcing researchers to estimate parameters through additional sensitivity analysis. As SEPMs are some of the most accurate models, modelers may be inclined to create additional variables, or to

enhance existing variables. Furthermore, an uncertainty analysis may be conducted to validate the results of their testing, and when other variables are added it becomes increasingly difficult to quantify the sources of uncertainty causing potentially misleading model readings [5]. On top of a potentially misread model (this is not exclusive to SEPMs, but is more likely due to their extensive data/parameter requirements) SEPMs require massive computing power, have extensive research necessities, and have laborious time commitments for runtime and proper analysis, which leads to a limited number of individuals or habitat patches that can be simulated and a demand for a more efficient modeling system.

Through an explanation of the two examples it will be demonstrated how graph theory can play a significant role in one of today's most discussed issues. Additionally, due to the study of the two examples that focus on the utilization of graph theoretic modeling for potential conservation purposes, it can be seen that graph theory informs conservationists; thus, it can be suggested that graph theoretic models should be further utilized for their abilities to produce similar results to more well-known models, as well as for their high levels of efficiency.

In the following sections the population models, with respect to their spatial and physical properties, will first be described. The necessary environmental and mathematical knowledge needed to understand the more complex graph theoretic measures used by each of the population models will then be provided. Concluding the paper will be the results of each study and why the results are important to future conservation efforts.

## **2 Model Introductions**

In this section the GBR and Wood Thrush population models will be discussed in further detail. To better understand the habitat models they will be described thoroughly by their spatial locations, assumptions, and parameters.

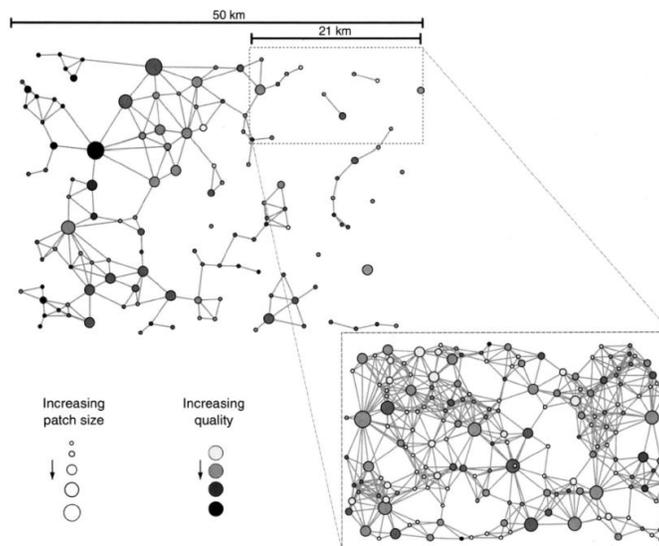
### **2.1 The Great Barrier Reef model**

Biodiversity is essential to the success of Australia's GBR system and is enabled by the oceanic and wind currents that carry fish and coral reef larvae between reefs. While there is a general acceptance to the importance of the larval transportation there is not a thorough understanding of its implications on reef functionality and management. The purpose of Kininmonth et al.'s research conducted on the 321 reefs within the central section of the GBR was done to examine and attempt to describe the network topology of the various reefs. To create a successful model of the Cairns section of the GBR a hydrodynamic simulation must be used due to the difficulty of measuring exact larval movements. The researchers used connectivity matrices from previous research, which measure the number of fish larvae that survived the voyage from their birth reef to their settlement reef, to power their Lagrangian hydrodynamic model: a model of spatial location and physical properties of the larvae and their movements. The Lagrangian hydrodynamic model was then used to simulate the releasing of the fish larvae (and some scattered coral larvae) from the 321 different reefs in the Cairns section of the GBR. Data was subsequently modeled and collected from 1967 to 1998 during the summer months when concentrations of larvae dispersal were at their highest [2].

It is possible for larvae to never leave the reef; however those larvae dispersals were ignored due to the authors' interests in inter-reef connections. The larvae were able to move freely in two-dimensional space with respect to tidal and wind currents, as they allow for variances in factors such as the timing of release, mortality rates, and precompetent and competent periods. These periods refer to the life stages of fish larvae: precompetent is the time before they undergo metamorphosis and competent is the time after metamorphosis in which they begin to move and eat [2].

## 2.2 Modeling of the Wood Thrush habitat

The focal landscape in Wake County, North Carolina is situated within the Research Triangle, an area where the three major universities (Duke University, North Carolina State University, and the University of North Carolina at Chapel Hill) perform much of their research as it is located between their borders, of the North Carolina Piedmont (region between the Atlantic Coast and the Appalachian Mountains) [5]. The first landscape in the model, Landscape 1, has an area of 1650 square kilometers with 126 patches, and the second, Landscape 2, is a subset of Landscape 1 with an area of 270 square kilometers and 172 patches. The graph model in Figure 1 shows the two landscapes with Landscape 1 on the upper left-hand side and Landscape 2 on the lower right-hand side [3]. To properly capture the landscape, high-resolution aerial photographs were taken where the landscape was broken down into pixels (much like a screen resolution uses pixels to create an image) and each pixel was marked by which of the six habitat categories best described it (sparse vegetation, highly reflective/developed, hardwood forest, pine forest, water, or mixed vegetation) [5]. The presentation of two nested landscapes was used to show connectivity at both the global and local level in order to properly determine the best sites for protection based on the attributes of size, quality, and connectivity.



**Figure 1.** Graph models of Landscape 1 and Landscape 2 with respect to habitat patch size and quality.

### 3 Environmental and Mathematical Concepts

In order to understand the graph models and their relevance to ecology, the necessary environmental and mathematical concepts will be introduced separately, and later their interconnections will be demonstrated.

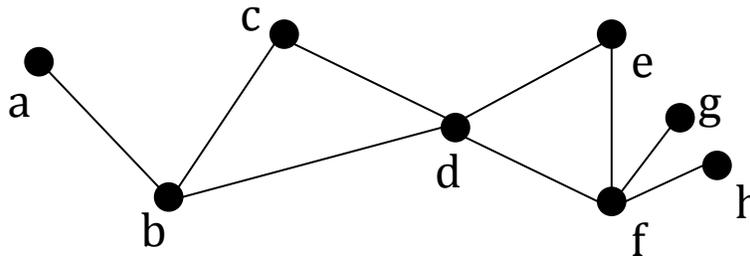
#### 3.1 Basic environmental concepts

Landscapes are evaluated at both a landscape scale and at an individual habitat patch scale to properly assess the level of connectivity within a landscape in order to select the “most valuable” habitat patches for conservation. A *habitat patch* is a specified area of land within a landscape that is known to have a species of interest living there. There are three different types of habitat patches, each having a dominant feature of size, quality, or connectivity. The first type is a *source patch*, which are patches that have populations with more births than deaths and where the focal species is more likely to emigrate from the patch rather than immigrate to it. The second, a *stepping stone patch*, is important to connectivity because it allows source patches to be linked to the smaller, or lower quality, patches within a network. When a stepping stone patch is the only link between different areas of a landscape its removal may break the entire habitat into multiple fragmented landscapes. Habitat fragmentation may be caused by natural or artificial disturbances, e.g., storms, shipwrecks, or human intervention. The third type of patch is a *persistent patch*, which is one that maintains its population throughout time, either through high levels of immigration or self-sustainment.

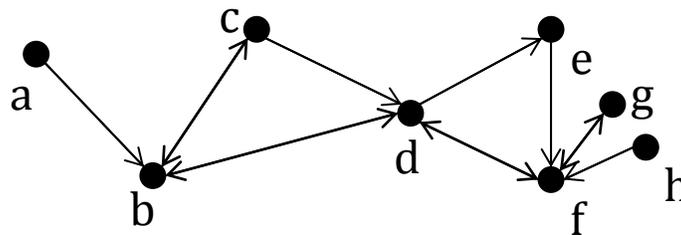
#### 3.2 Basic graph theoretical concepts

Some preliminary definitions, which may mostly be found in [6], are essential to understand the graph theoretic models. A *vertex*, also referred to as a *node*, represents a point in space and is generally denoted by  $i$  or  $j$ , where  $n$  is the total number of nodes. An *edge* connects any two nodes and creates a vertex pair,  $V(i, j)$ . The number of possible vertex connections among  $n$  nodes is  $\binom{n}{2} = \frac{n*(n-1)}{2}$ . *Diameter* is the longest minimum path length that exists between any pair of vertices in a network. *Degree* refers to the number of edges extending from a single vertex and is here denoted by  $k$ , or specifically  $k_i$  for a given vertex labeled  $i$ . A *graph* is then a collection of vertices and edges. A *walk* is a sequence from node to node along a graph’s edges measured by the number of steps taken along the way. Hence, a *path* is a walk where no node is visited twice; it is said that a graph is *connected* if there is a path between each pair of nodes. If a graph is disconnected it contains more than one connected component, where a *component* is a subset of the given graph where no walk exists between each connected subset. One way to disconnect a graph is through the removal of a *cut-node*, a node whose removal disconnects the graph. Cut-nodes are important to this research because they create graph components. Directional graphs will also be considered; they are graphs where an edge may only be traversed in one direction or the flux (movement) between two nodes is different depending on which direction an edge is travelled. For example, in a directional graph, edge  $ij$  may be traversed, but

edge  $ji$  may not be traversed, whereas in a “basic” graph the edges  $ij$  and  $ji$  are assumed to be equivalent. A *network* is then a directional graph with weighted nodes and edges where the weight of a node may correspond to the object of representation’s spatial size and the weight of an edge may correspond to the strength of the connection between two nodes.



**Figure 2.** The graph,  $G$ , contains nodes  $\{a, b, c, d, e, f, g, h\}$  and edges  $\{ab, bc, bd, cd, de, df, ef, fg, fh\}$  where  $d$  is a cut-node with degree  $k_d = 4$  whose removal results in components  $\{a, b, c\}$  and  $\{e, f, g, h\}$ . One walk in  $G$  would be  $\{ab, bc, cd, db\}$ , which has a distance of 4 steps. This walk would not be a path because node  $b$  is visited twice.



**Figure 3.** The directional graph,  $G$ , contains nodes  $\{a, b, c, d, e, f, g, h\}$  and edges  $\{ab, bc, cb, bd, db, cd, de, df, fd, ef, fg, gf, hf\}$ .

### 3.3 Basic environmental concepts in relation to graph theory

A node in Kininmonth, et al. represents an individual reef and an edge displays the connection of two reefs through fish larvae dispersal. In the model given in the article, a directed graph is used assuming that larvae can flow in only one direction. In the construction of the network, reefs were modeled with their respective spatial features of location, perimeter, and area. Inter-reef connections were assigned their attributes of strength, distance, and the direction in which larvae flow. In reef models it was assumed that dispersal occurred with uniform density along the reef edges causing the strength of the connections to be a function of size and shape. Components are then representations of habitat fragmentation.

Some application-specific definitions from Minor and Urban [3] are needed in order to understand the Wood Thrush population models in the paper as well. First, a node in Landscape 1 is any habitat patch bigger than 50 hectares where one hectare (ha) is equal to 10,000 square meters. A node in Landscape 2 is any patch bigger than five ha, revealing that Landscape 2 (a subset of Landscape 1) is a finely grained landscape and will demonstrate higher levels of connectivity, whereas Landscape 1 is more coarsely grained allowing for the appearance of habitat fragmentation, which is visible in Figure 1. The distance between a habitat’s boundaries

defines edges; and patches are connected if they are within 1,500 meters of each other due to research determining dispersal distances.

Given a network model, the next step is to determine its characteristics with the definitions that follow. Small-world networks are characterized by landscapes that contain shortcuts between two distant habitat patches allowing for shorter diameters relative to the number of nodes in the graph. Shortcuts are generally caused by natural disturbances such as strong winds carrying birds further than originally anticipated, or by human intervention, i.e., organisms travelling on boats [7]. An example of a shortcut would be in Figure 2 if nodes  $a$  and  $e$  were connected. Landscapes with small-world characteristics tend to be vulnerable to random disturbances since habitat patches can be reached with minimal edge traversing caused by high levels of clustering and short path lengths.

A scale-free distribution is when there are a small number of high degree nodes with the majority of nodes being low degree nodes meaning that many nodes may share a mutual neighbor while not being neighbors themselves. One example being the Internet, where search engines such as *Google* are connected to most websites, but very few of those websites are connected to each other.

## 4 Graph Theoretic Measures

In order to understand the graph models and their relevance to ecology, the necessary environmental and mathematical concepts will be introduced separately, and later their interconnections will be demonstrated.

### 4.1 Graph theoretic measures in relation to the Great Barrier Reef model

Within each model various graph theoretic measures are used to analyze the setting. Since both applications use their own measures they will be considered in order of appearance through each application. The first application is the Cairns Section of the GBR, where Kininmonth, et al. presents the following quantities: a *triangular linkage* is a triangle that is formed starting at the target vertex. The *clustering coefficient*,  $CI$ , for a weighted directional network is defined as the sum of the triangular linkages  $\vartheta_i$ , where  $i \in V_1, \dots, V_n$ , within a set of neighbors for a selected vertex divided by the maximum possible neighbor linkages:  $CI = \frac{1}{n} * \sum_{i=1}^n \frac{\vartheta_i}{k_i * (k_i - 1)}$ . The *average minimum path length*,  $L = \frac{1}{n * (n - 1)} * \sum_{i=1}^n \sum_{j=1}^n L_{Min}(i, j)$ , is the shortest number of edges in a shortest path,  $L_{Min}(i, j)$ , between every vertex pair in a graph, averaged over the number of possible vertex connections. *Diameter* ( $D$ ) is the longest minimum path length that exists between any pair of vertices in a network. *Closeness Centrality* for a vertex  $i$  ( $CC_i$ ), is the sum of minimum path lengths connecting vertex  $i$  to all other vertices  $j$ . A low  $CC_i$  value corresponds to isolation of a vertex, whereas a high  $CC_i$  value relates to a small diameter for each vertex pair and leads to a higher level of connectedness among vertices. This causes an elevated level of contagion (the ability for disease to spread easily through a network and potentially destroy that network). *Betweenness Centrality* for a node  $k$  ( $BC_k$ ) is the proportion of minimum paths that

connect vertices  $i$  and  $j$  and pass through the vertex  $k$ . A high  $BC$  value indicates that a vertex is a stepping stone patch.

To determine the strength of an edge, each edge was assigned an edge weight value ( $w_{ij}$ ) if the strength of a connection, or amount of interaction between two vertices  $i$  and  $j$  is known. This describes the amount of dispersal between two reefs. Strength indices can then be defined for each vertex  $i$  by  $s_i^{IN} = \sum_{j \in v(i), j \in E_{ji}} w_{ji}$  and  $s_i^{OUT} = \sum_{j \in v(i), j \in E_{ij}} w_{ij}$  where  $v(i)$  is the vertex's set of neighbors,  $E_{ji}$  is the set of inbound edges, and  $E_{ij}$  is the set of outbound edges. The average of the inbound and outbound strengths in the graph can also be observed with the equations:  $S_i^{in} = \frac{1}{n} \sum_{i=1}^n s_i^{IN}$  and  $S_i^{out} = \frac{1}{n} \sum_{i=1}^n s_i^{OUT}$ .

Some small-world networks may be characterized by the scale-free distribution where  $P(k)$ , the number of vertices with  $k$  connections, is represented by the power law,  $P(k) \sim k^{-\gamma}$ , for some constant  $\gamma$ . The power law, similar to a scale-free distribution, demonstrates that the majority of vertices are weakly connected and small, yet a significant number are very strongly connected. However, it has been found that the power law is not entirely consistent at representing a scale-free network. Therefore the size-rank measure is used when the power law is not convincing of the scale-free property. The size-rank measure is defined as  $R(k) = cY_k^{-\gamma}$ , where  $c$  is a fixed constant, and  $\gamma$  is a scaling index. The value of  $R(k)$  may be used to rank the degree of  $y_k$  from a finite sequence  $y_1 \geq y_2 \geq \dots \geq y_n$ .

## 4.2 Graph theoretic measures in relation to the Wood Thrush population model

To better understand the graph theoretic measures involved in the comparison of graph models to SEPMs in Minor and Urban's research this subsection will review how they are defined. The first parameter related to the nodes of a graph is called the quality-weighted area ( $QA$ ). This is the patch quality multiplied by the patch size (in ha) that relates to the reproductive potential of a patch such that higher  $QA$  means a greater number of offspring and vice versa. The highest quality patch corresponds to the maximum reproductive potential for its size and vice versa. Patch quality is represented by the distance from a non-forest edge for every pixel in a habitat patch following an eight-neighbor rule (where pixels have at least one corner in common) and is in the range from 0 to 1. Quality can be affected by vegetation type and proximity to human development. The next measure, flux, corresponds to the amount of movement from one patch to another along the edges of a graph and is defined by  $Flux_{ij} = QA_i \times P_{ij}$  where  $P_{ij}$  is the probability of dispersal between two patches and is modeled by a negative exponential decay function:  $P_{ij} = \exp(\theta \times d_{ij})$  where  $\theta$  is the distance-decay coefficient ( $\theta < 0.0$ ) that determines the curve's slope and  $d_{ij}$  is the distance between patches. The flux equation demonstrates that the influx from patch  $i$  to patch  $j$  will not necessarily be equal to the outflux from patch  $j$  to patch  $i$  since it depends on the quality-weighted area of the first patch. A global graph goodness metric was also computed in which the influx and outflux were averaged. Finally, betweenness values were computed for each node, which helps determine a patch's value as a stepping-stone.

SEPMs are a more biologically accurate modeling system that can model the behavior of a focal species in a specific environment and is often used by conservationists to determine

potential consequences of habitat disruption and for considering what land areas should be conserved. The SEPMs requires all of the same data as the graph theoretic model; however, there are nine extra parameters that must be considered, and that can be observed in the following table, Table 2, which is being used with permission from Minor and Urban [3]. From the parameters and two initialization processes (random and single patch initialization) output metrics were created and are listed in Table 3 below, which is being used with permission from Minor and Urban [3]. Whereas SEPMs offer more biologically accurate outputs due to their complicated parameters and output metrics, their level of complexity can lead to complications from their high potential for error. For instance, if there is a small error in the calculation of any of the parameters, it will then have a “snowball effect” when the model is created, and increase the amount of error when the output metrics are calculated from the inaccurate model. The graph goodness metric was also computed for the SEPMs where the source strength and persistence were averaged due to their nearly equal values when creating a reserve, which cause the averaging of their comparative metrics influx and outflux for the graph goodness measure.

TABLE 2. Parameters used in SEPMs (spatially explicit population models).

Parameter	Value	References
Maximum nest parasitism rate	75%	Donovan et al. (1995), Fauth (2001)
Maximum nest predation rate	65%	Hoover et al. (1995), Brawn and Robinson (1996)
Annual adult survival	0.65	Powell et al. (2000), Roth et al. (1996)
Annual juvenile/floater survival	0.3	Anders et al. (1997)
Clutch size	2.5 eggs	Trine (1998)
No. broods attempted annually	2	Roth et al. (1996)
Territory size	1 ha	Twomey (1945), Weaver (1949)
Dispersal distance	1.5 km	Anders et al. (1998)
Juvenile/floater site fidelity	0.10	Estimated data
Breeder site fidelity	0.90	Estimated data

TABLE 3. Output metrics from the SEPM.

SEPM output metrics	Definition
BL (number of breeders on whole landscape)	From the single-patch initialization version of the model, the number of birds on the landscape after 100 years following initialization from the focal patch.
BP (number of birds on each patch after 100 years)	From the random initialization version of the model, the number of birds on the focal patch after the model reaches equilibrium.
CV (coefficient of variation of patch population over last 10 years of model run)	From the random initialization version of the model, the coefficient of variation of the patch population in the last 10 years of model runs.
YO (how many of the last 10 years the patch was occupied)	From the random initialization version of the model, the number of the last 10 years that the focal patch was occupied.
Source strength	$(\text{rank BL} + \text{rank BP})/2$
Persistence	$(\text{rank CV} + \text{rank YO})/2$
Overall goodness	$(\text{source strength} + \text{persistence})/2$

## 5 Graph Theory Applications to the Modeling of the Great Barrier Reef

Now that the characteristics and measures of the models have been reviewed, we will see how they can inform environmental decision making, beginning with the GBR study. The research done in Kininmonth et al.’s paper aids in demonstrating the usefulness of graph theory in relation to wildlife conservation. The resulting study demonstrated that the average minimum path lengths ranged from 2.2 to 3.9, average *CI* in the range from 0.26 to 0.31, average degree ranging from 67 to 128, and edge count in the range of 10,860 to 20,554. From these results, Kininmonth

et al. concluded that the GBR network is a small-world network due to the comparison to some of the other constructed small-world networks such as the Watts-Strogatz network and the Forest Fire network using their standards. This result can be seen in Figure 4 where clustering coefficients were graphed against average minimum path lengths.

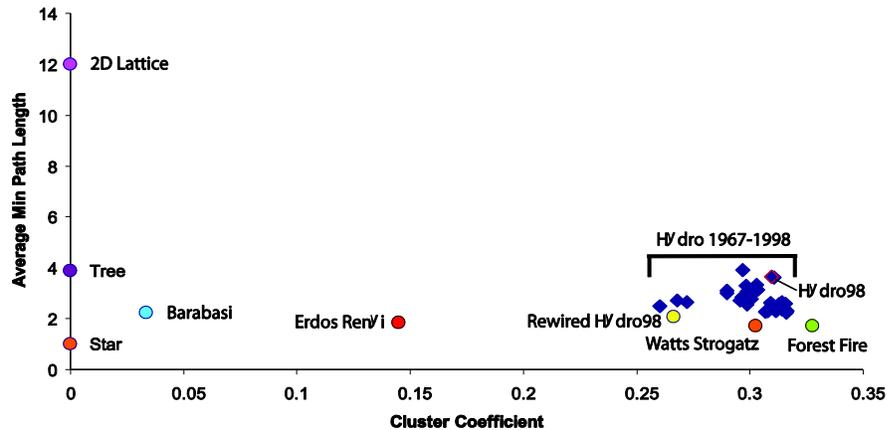


Figure 4. Demonstration of small world properties (image used with permission from Kinninmonth et al.).

The model also appeared to be a large diameter network with only a small number of reefs acting as stepping stones [2]. It was found through statistical analysis that the closeness measure demonstrated a normal distribution indicating that some reefs were isolated and others were strongly connected. From both the size-rank measure and degree frequency, the GBR network was deemed not to be scale-free relative to degree distribution. The results lead to an interesting contradiction where the small-world properties demonstrate a large amount of connectivity, but the large diameter property represents a lower level of connectivity.

In a model of the 1998 graph, edges were removed strongest first, randomly, and weakest first, in order to observe how well connected the network is to determine whether the large diameter property or small-world properties were more prevalent. Through this edge removal process, it was discovered that 98% of the edges had a weight less than 0.14 and the weaker edges in this model tended to be the longer edges distance-wise and held the network together. The strongest edges first and weakest edges first methods accentuated the small-world properties, which importantly demonstrates that the GBR network is a very strongly connected network where reefs can be reached with minimal traveling. The random removal method demonstrated that the GBR is protected from random disturbances so long as there is no regional decrease in the capability for larvae to migrate long distances [2].

The importance of the reefs is not based on the amount of larvae circulating to and from each reef, rather on their centrality metrics. For each reef with a measure of high  $CC$ , low  $CC$  and high  $BC$ , the reefs with the highest rank were identified and studied through the 32 years to show that the reefs were consistent for their respective centrality metrics through time [2]. This demonstrates that conservation efforts should be biased towards these reefs because of their consistency and importance as highly travelled reefs.

While the number of connections is not scale-free, Kinninmonth et al. determines that “the distribution of reefs with respect to the number of larvae settling or migrating to another reef can be described as scale free,” [2] which is due to the large number of reefs that are strongly connected. Until proper parameters for larvae dispersal are developed the GBR network could be a series of isolated communities or it could be one extensive network. The small-world pattern remains questionable and shows that more field research needs to be completed to solidify an understanding of how the GBR network works. Without comprehension of the metapopulation structure there can be no understanding of the functionality of the coral reef system, and so the graph theoretic modeling system remains an asset to conservationists to decide where in the GBR protected areas should be, even if its level of accuracy is unknown.

## 6 Comparison of Graph Theory to SEPMs

In comparing the graph model to the SEPMs, outflux was matched with source strength and influx was paired with persistence. On the graph of ranked outflux versus ranked source strength there was a strong positive correlation ( $\rho = .86$  for Landscape 1 and  $\rho = .87$  for Landscape 2). This happened because both metrics identify patches that contribute a large number of birds to the landscape and any variability was due to the relationship between outflux and the number of breeders on the landscape. For a more visual representation, a graph model, shown in Figure 5, was composed to show the top ten patches chosen for both outflux and source strength. The image on the left-hand side reveals an overlap of seven patches in Landscape 1 and eight patches in Landscape 2, and indicates their strong positive correlation. On the graph of ranked influx versus ranked persistence, there was still a strong positive correlation, but not as significant as in the previous comparison where the patches with lower values appeared to have stronger relationships; this is demonstrated by a high positive correlation of  $\rho = .87$  for Landscape 1 and  $\rho = .73$  for Landscape 2. In the middle graph model, two nodes were in both the top ten patches for persistence and influx in Landscape 1 and only four patches were the same for Landscape 2, which did not demonstrate the correlation values mentioned, and is largely due to the relationship between influx and the coefficient of variation. In the comparison of the graph goodness and SEPM goodness metrics there was a correlation value of  $\rho = .93$  for both landscapes. The result demonstrates how “good” the metrics perform and verifies the statement that source strength and persistence will have virtually equal values. In another graph model where both of the goodness measures were compared, five of the top ten patches were the same for Landscape 1, and eight of the top ten patches were the same for Landscape 2.

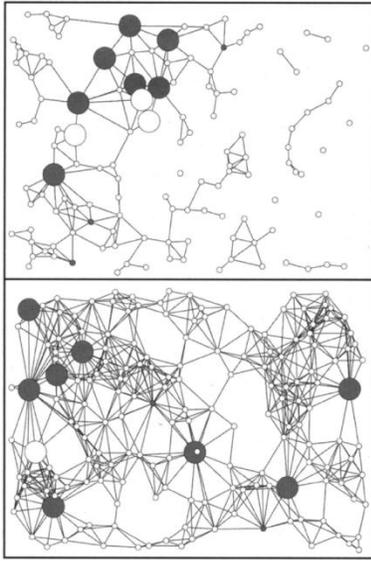


FIG. 3. A comparison of graph outflux and the SEPM metric "source strength." The large landscape is shown in the upper panel, and the small one in the lower panel. The top 10 patches for graph outflux are shown as dark circles, while the top 10 patches for source strength are indicated by the large circles. Patches that are both dark and large are in the top 10 for both models.

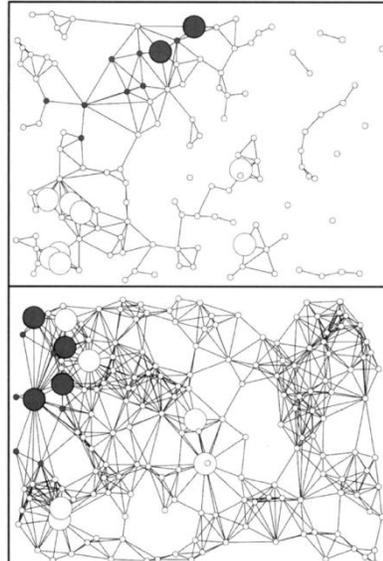


FIG. 5. A comparison of graph influx and the SEPM metric "persistence." The large landscape is shown in the upper panel, and the small one in the lower panel. The top 10 patches for graph influx are shown as dark circles, while the top 10 patches for persistence are indicated by the large circles. Patches that are both dark and large are in the top 10 for both models.

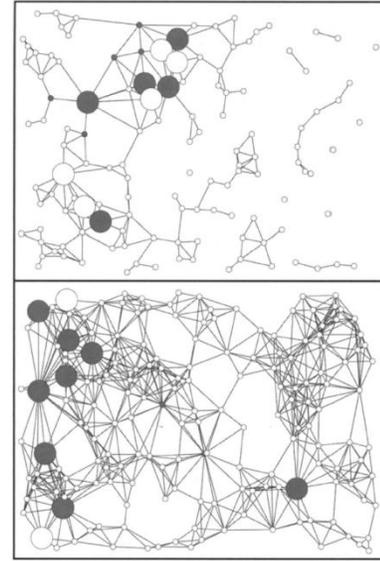


FIG. 7. A comparison of overall graph "goodness" and SEPM "goodness." The large landscape is shown in the upper panel, and the small one in the lower panel. The overall top 10 patches for the graph model are shown as dark circles, while the overall top 10 patches for the SEPM are indicated by the large circles. Patches that are both dark and large are in the top 10 for both models.

**Figure 5.** Comparisons of SEPMs and graph metrics for Landscape 1 and Landscape 2 (used with permission from Minor and Urban) [3].

## 7 Conclusion

Extensive research has been performed for comparing different species when determining the best patches for conservation in a landscape. However, it poses further problems due to the complexities in behavior. Although there are always new issues presenting themselves, there are almost as many proposed answers that claim to be able to improve the quality of graph theoretic modeling. Through research on the GBR, interactions within this network become observable in a manner that has never been seen before. The models allow for the understanding of the functionality of the GBR, and show the most important areas of conservation by the consistent measures of high betweenness centrality, high closeness centrality, and low closeness centrality.

Comparing the metrics of graph theoretic models to the SEPMs was not to determine which model and respective metrics was better, but to test the usefulness of the graph theoretic models; this work was successful because the graph metrics performed similarly to the SEPM metrics. The result was noticeable through the overall positive correlations the SEPM metrics and the graph theoretic metrics. The graph theoretic model also proved beneficial in its minimal data requirements for creating models with a lower potential for error when creating parameters such as the ones required for SEPMs

Due to the analysis of the two examples of the utilization of graph theoretic modeling for conservation purposes the following understanding is allowed: when time, money, and knowledge of a species are minimal graph theory can act as a proxy for the SEPMs without causing a high potential for error. It can then be seen that graph theory can inform conservationists, and therefore graph theoretic models should be utilized more often, due to their

abilities to produce similar results to more well-known models and for their high levels of efficiency. This allows for better reserve site selection for a focal species improving their long-term health and allows for enhanced understandings of metapopulation structures in a landscape and the importance of the connectivity of habitat patches.

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