Quantitative Data Extraction using Spatial Fourier Transform in Inversion Shear Interferometer

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Quantitative Data Extraction using Spatial Fourier Transform in Inversion Shear Interferometer

A Thesis
Submitted to the Faculty
of
Rose-Hulman Institute of Technology
by
Yanzeng Li
In Partial Fulfillment of the Requirements for the Degree
of
Master of Science in Optical Engineering
August 2014

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Quantitative Data Extraction Using Spatial Fourier Transform in Inversion Shear Interferometer

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ABSTRACT

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May 2014
Quantitative data extraction using spatial Fourier transform in inversion shear interferometer
Thesis Advisor: Dr. Scott R. Kirkpatrick

Currently there are many interferometers used for testing wavefront, measuring the quality of optical elements, and detecting refractive index changes in a certain medium. Each interferometer has been constructed for a specific objective. Inversion shear interferometer is one of them. Compared to other interferometers, it has its own advantages, such as only being sensitive to coma aberration, but it has some limitations as well. It does not allow use of phase shifting technique. A novel inversion shear interferometer was invented using holographic lenses. By using the spatial carrier method, phase information of the wavefront was extracted. The breakthrough of the novel technique includes real-time quantitative analysis of wavefront and high stability in turbulent conditions.
In this thesis, I discuss the operating principles for the new inversion shear interferometer, and discuss the process of quantitative analysis after integrating spatial Fourier transform analysis. I also present how to exploit the set of holographic lenses to setup the inversion shear system. The advantages and disadvantages of the novel inversion shear interferometer are summarized, and some solutions for improvement are also suggested.
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LIST OF ABBREVIATIONS

AS       Aperture Stop
AW       Aerrant Wavefront
CR       Chief Ray
EnP      Entrance Pupil
ExP      Exit Pupil, 21
FWHM     Full Width at Half Maximum
OPD      Optical Path Difference
OPL      Optical Path Length
RW       Reference Wavefront
1. INTRODUCTION

1.1 Background

With the development and advances in sophistication of optical products, the demand for making high precision measurements and testing has increased. Particularly in optical instruments and microscopic devices (such as chips and waveguide derivative products) manufacturing sectors, the majority of working systems can tolerate instrumental errors on the order of only a few micrometers, which could be introduced not only by system alignment deviations, but also by errors coming from each individual component [1]. Furthermore, high precision components are extremely sensitive and susceptible to damaged due to external pressures and strains as well, even though such pressures and strains are subtle. Therefore, the traditional method of measuring by observing fringes is not suitable for the aforementioned components. In addition, there are other abstract properties of the objects that need to be tested, such as aberrations of the lenses, the refractive index changes and measurement of three dimensional data of microscopic structure, all of which cannot be obtained directly by conventional measurement [1]. Fortunately, such detections have been achieved using interferometry. Many scientists and engineers have been working in this field in last fifty years and a number of techniques based on interferometry have been developed during this time. They have successfully been used for non-contact measurement in industry and for making Nondestructive Testing (NDT) [2].
Shear interferometry is a member of the interferometry family. It is a powerful way to test the quality of the wavefront. The quality of the wavefront means what kind of aberrations the wavefront carries. Lateral shear interferometry is a typical example, which provides high precision measurement and keeps the test process simple [3-5]. Besides lateral shear interferometry, other types of shear interferometry have been developed, such as radial shear interferometry, inversion shear interferometry and reversal shear interferometry [6, 7].

1.2 Theory of Interferometry

In interferometry the wavefront to be tested is made to interfere with a reference wavefront [8]. However, in all these methods, the inherent properties of light is exploited, namely the phase information. Phase depends not only on the path the light travels, but also on the propagation medium. Traveling different paths or traveling in mediums with different refractive index can cause coherent light to have different optical path length (OPL) because OPL is the product of refractive index $n$ and path length $L$. The reference and object interfere to form dark and bright fringes, which occurs because of the phase difference. In optical measurements, such phase difference is caused by defect factor changing the object wavefront. Simply speaking, the interference pattern could be treated as a media recording the information of the object wavefront indirectly.
Interferometry measurements are non-contact techniques that protect the object from being damaged. Thus, two main factors, high accuracy and non-contact detection, make interferometry a possible tool for testing in industry.

1.2.1 Coherence

Light is a fundamental component for optical interferometry, but light by itself does not have the capability of producing interference patterns. Parameters such as frequency, direction of oscillation and phase difference are collectively known as coherence properties of electromagnetic radiation. To cause interference, two beams of light that are coherent are made to superpose by simple optical arrangement.

Briefly speaking, spatial coherence implies that the size of the real light source should be as small as possible to guarantee that light waves from the source can provide relatively constant phase differentials in space. In white light interferometer, for example, a combination of condensers and apertures are used to decrease the size of white light in order to enhance its spatial coherence [1]. Temporal coherence is the other important factor which is strongly related to the ability of light to exhibit interference effects when two lights are separated by a distance. Temporal coherence represents average correlation between the value of a wave and itself delayed in time [9]. As the delayed time reduces, a wave contains larger range of frequencies and becomes more difficult to interfere with itself at a different time. Laser is the light source with extremely narrow bandwidth, which is one of the reasons of why most interferometers use a laser as a light source.
1.2.2 Classification of interferometers

Interferometers can be classified and categorized into different groups: by optical paths such as double and common paths and by the splitting beam method such as wavefront splitting and amplitude splitting. The theory and basic principle behind this thesis is demonstrated in detail below.

The interferometer which is classified as a double path interferometer or common path interferometer depends on how the original light source is divided. Generally, for double path interferometer a beam splitter is a common tool for splitting one beam into two exactly same waves in different directions. In these two divided waves, one is called the reference wave, and the other is called the object wave. The difference between them depends on their optical paths and fluctuations that arise between the two beams. In a Twyman-Green interferometer, for example, the reference wave goes back and forth from a perfect mirror and remains flat, but the object wave suffers from transferring back and forth through the test optical system and is deviated from being flat [10, 11]. The object wave is combined with the reference wave and the phase difference between them creates an interference pattern which then can be interpreted to determine the phase difference [12].

In a common path interferometer the reference and object waves travels along the same path. Examples of common path interferometers are: the Zernike phase contrast interferometer, Zero-area Sagnac interferometer, inversion shear interferometer and
lateral shear interferometer [13]. For these interferometers, even though their reference and object waves are traveling along the same path, their directions may be the same or be opposite or can have different polarization.

Both the double path interferometers and the common path interferometers have their own advantages and needs for different applications. Due to high sensitivity to phase shifts and path length changes between the reference and object arms, double path interferometers are widely applied in science and industry for measuring small displacement, refractive index changes, and surface irregularities [1]. Compared to the double path interferometer’s high sensitivity, the common path interferometers are more applicable in harsh conditions, such as vibrations, because of their excellent resilience to environmental agitation. [14].

1.3 **Lateral shear interferometer (LSI)**

Lateral shear interferometry is an important technique in the interferometry field for measuring optical components and systems. It has diverse applications in multiple fields, such as the study of gas and liquid flow, microscopic structure detection, and intermediate refractive index changes [15]. At the same time, due to its high resilience to environment and arrangement feasibility, lateral shear interferometry has had more researchers using it for making high precision measurements.
The basic method to fabricate a lateral shear interferometer includes three steps: duplicating the test wavefront, displacing it laterally, and making the displaced wavefront to interfere with the original test wavefront. These three simple steps give researchers an ample space to design lateral shear interferometry with diverse optical components. One can use either a single optical element or multiple optical elements.

1.3.1 Basic theory

An ideal wavefront is a plane wave. After transferring through a test optical component, the plane wavefront will be distorted because of aberrations $W(x, y)$, where $(x, y)$ are the coordinates of any point. Aberrations $W(x, y)$ is usually described as a polynomial function with deviations from an ideal spherical beam. In other words, the initial value of the wavefront can be denoted as $W(x, y)$. Following the three aforementioned steps, the duplicated wavefront is also expressed as $W(x, y)$, but is displaced in the y direction by an amount $S$. It is rewritten as $W(x, y + S)$, where $S$ can be either positive or negative. Since there two wavefronts interfere, the optical path difference $\Delta W$ between them can be expressed as [1]

$$\Delta W = W(x, y) - W(x, y + S)$$

From Eq. (1), the deviations of the test wavefront from a perfect sphere can be extracted from $\Delta W$. If the shear $S$ is zero, $\Delta W$ is zero consequently, which means that optical path difference and aberration cannot be detected in the wavefront area. Non-zero
shear is needed. However, large shear results in mass fringes in the shear area. There are two ways of increasing the number of fringes in the sheared area: i) increase the shear and ii) increase the tilt angle.

In my experiment, the tilt angle is controlled such that the interference fringes work as the information carrier [15]. In addition, the number of the interference fringes can be large or small by controlling the angle between the two beams. Also, large shear leads to high frequency fringes as well, which results in high sensitivity. But if the specimen is a microscopic structure, for example a wafer surface, a lot of detail is lost at the boundaries because of the amount of shear (S).

Theoretically, the maximum number of interference fringes can be expressed [1]

\[ \Delta W = n\lambda \]  \hspace{1cm} (2)

In this equation \( n \) is the order of the interference fringe and \( \lambda \) is the wavelength. Eq. (2) could be written in another way when the shear \( S \) is small [1].

\[ \left( \frac{\partial W}{\partial y} \right) S = n\lambda \]  \hspace{1cm} (3)

From Eq. (3), the aberration information in a lateral shear interferometer is an angular measure. This information can be obtained more exactly when the shear \( S \) is
approaching zero, but the sensitivity of the whole system reduces at the same time. Therefore, a proper value of shear should be adjusted for specific conditions.

1.3.2 **Optical devices in lateral shear interferometer**

Many optical components and optical systems have been used in lateral shear interferometers. Some of them are simple and some of them are complicated, depending on their functions. A shear plate is the simplest optical device for lateral shear and is the standard tool for checking beam collimation (shown in Fig. 1 [16]). A shear plate is made from one piece of thick glass with both its sides polished to be parallel to each other. Using both the thickness between the two sides and the partial reflection, the shear plate easily generates a reflected original wavefront in the first layer of glass and duplicates another one at the second layer simultaneously. In the experimental arrangement, the shear plate is inserted in the path of the beam to be tested and then the interference pattern is obtained.

A shear plate is generally used for quantitative analysis of the wave. To make accurate measurements, generally a phase shifting interferometric technique is used. However, for dynamic changes, a shear plate is not an ideal tool. Therefore, an alternate method of extracting the phase was one of the goals of my thesis.
In this thesis, I will demonstrate how a holographic lens can be used to not only create lateral shear but also be used for information extraction. The reason for choosing
holographic devices is because it is light weight and easy to use. The technique also has the flexibility for adjusting the lateral shear depending on the experimental need. The main feature that differentiates the holographic lens from other lateral shear optical devices is that the holographic lens can be used to test in a collimated wavefront. Inserting a holographic lens with double frequencies into the path of a test beam allows the original and duplicated beams to emerge right behind the holographic lens. The amount of the shear between the beams is determined using a frequency difference. Additional shear can be introduced by displacing the two holographic lenses in the lateral direction. This method for obtaining lateral shear measurements largely enhances the practical use of this technique by greatly simplifying the system.

The other remarkable property of the holographic lens is its ability of splitting white light into the visible spectrum. This property is used in applications such as the solar energy concentrator and the white light lateral shear interferometer. The theory of dividing light into different wavelengths is based on diffraction. The solar energy concentrator uses holographic lenses to work as a spectrum splitter, by which far infrared is diffracted away from the photovoltaic system, reducing the heat on the solar cell [17-19]. The holographic lens helps photovoltaic systems maximize the efficiency of conversion of solar energy to electrical energy and minimizes energy loss at the same time.
In white light lateral shear interferometer, a holographic lens acts as a spectrum splitter as well [20]. Due to low spatial and temporal coherence of white light, strict length matching is required. When white light falls on the holographic lens, the emerging light is no longer white light, but is split into its spectrum. The reason is the same as the above one: different wavelength has different diffraction angle according to Bragg’s law. Since both the lenses have identical diffraction orders, the wavelengths are made to match and interfere with itself.

1.4 Motivation of the thesis

As I mentioned previously, lateral shearing instrument is the most popular shearing interferometer, especially working with a holographic lens. However, other types of shearing interferometer are equally powerful such as an inversion shear interferometer. Both lateral shear interferometer and inversion shear interferometers have their own advantages and disadvantages. The motivation of this thesis is to construct an inversion shear interferometer. Secondly, another goal is to describe a method to introduce spatial carrier fringes and use the Fourier transform method to extract quantitative data.

Inversion shear interferometry is an exceptional case of rotational shear interferometer. They both obtain an interference pattern by generating two identical wavefronts with one of them rotated with respect to the other about their common optical axis [1]. The only difference between them is the rotation angle. In a rotational shear
interferometer, the rotation angle is any value of the angle less than 180 degrees; but in an inversion shear interferometer, the rotational angle is equal to 180 degrees [21]. The relationship of their rotation angle has been shown in Fig. 2 [1].

Because of the rotation characteristic, rotational shear interferometer is only sensitive to non-symmetric aberrations, namely astigmatism, coma and tilt [1]. Murty and Hagerott in 1966 had presented that the sensitivity for astigmatism and coma vary with the rotational angle [22]. When the rotational angle is equal to 180 degrees, inversion shear interferometer is only sensitive to coma and tilt [21]. This specificity of sensitivity is an advantage. For example, when symmetric aberrations are dominant over non-symmetric aberrations in the optical system under test, inversion shear interferometer would be a better test for non-symmetric aberrations than ordinary interferometers but the disadvantage is its interference pattern analysis. During traditional image analysis, the interference pattern suffers from the need to find fringe centers and the result has a tradeoff between precision and the number of data points [1]. This disadvantage has been solved by using phase shifting analysis.
In this thesis, I present a new method of combining the advantages of a lateral shear and inversion shear interferometer to develop a new inversion shear interferometer with a holographic lens. Two identical holo-lenses and two holo-gratings with a slightly varying frequency are the main optical components in this novel inversion shear...
interferometer. Utilizing the different diffraction angles between the two holo-gratings, which is decided in the process of fabricating these holo-gratings, makes the emergent wavefronts have a relative lateral shear and generate spatial frequency carriers at the same time. Spatial Fourier transform in the interference pattern is performed via MATLAB. I also show in this thesis how to fabricate these holographic optical elements since they are critical parts of this technique. Based on consideration of having a compact optical system, limitation of the setup of fabricating holo-gratings and resolution of the camera capturing the interferogram, the incident angle and the diffraction angle have to be chosen with specific values. Meanwhile, a high diffraction efficiency of each holographic optical element should also be obtained so as not to lose optical power [23].
2. THEORY

2.1 Inversion shear interferometer

The dominant on-axis aberration in lenses is spherical, but other aberrations, like astigmatism and coma, play a role in off-axis situations. Therefore, some terms in on-axis analysis, such aberrations are generally ignored during wavefront testing. However, if spherical aberrations are removed from the system, the sensitivity of the whole system to other aberrations becomes increasingly important.

There are different orders of aberrations, such as primary aberrations and higher order aberrations. They can be distinguished by looking at the power exponent and subscript of their coefficient. Also, the expression and the calculation methods differ between different orders or aberrations. The aberration polynomial describes the wavefront deformations from a perfect sphere. Aberration polynomials for primary aberrations can be expressed as [24]:

\[ W(x, y) = A(x^2 + y^2)^2 + B(x^2 + y^2) + C(x^2 + 3y^2)^2 + D(x^2 + y^2) + Ey + Fx + G \]

where \( A = \) spherical aberration coefficient

\( B = \) coma coefficient

\( C = \) astigmatism coefficient

\( D = \) defocusing coefficient

\( E = \) tilt about the x axis
\[ F = \text{tilt about the y axis} \]

\[ G = \text{constant} \]

This expression was given by Kingslake [24]. From this expression, one can roughly show that spherical and defocusing aberrations belong to a rotationally symmetric aberration, but coma and astigmatism are non-symmetric aberrations. The above expression has limitations because it can only shows primary aberrations. Besides primary aberrations, there are high-order aberrations. In order to take everything into consideration, power-series expansion is used here.

Because the experimental setup is a rotationally symmetric system, the cross-section diagram and 3-D schematic of the whole system is shown in Fig. 3 [25] and Fig. 4 [26], which is not an exact structure of the optical system developed in this thesis because it is a simplified model.
Figure 3: Cross-section diagram of optical system [25].
Figure 4: 3-D basic diagram of optical imaging system with present of aberration [26].
Observing an arbitrary point $P$ in the object plane (Fig. 3), its CR (chief ray) shows that its image-forming goes all the way through the center of EnP (Entrance Pupil), AS (Aperture Stop), and ExP (Exit pupil) until reaching a Gaussian image of a point $P'$ at the image plane. This diagram presents that how an ideally perfect imaging system without any aberration looks. The track of the CR demonstrates that the corresponding wavefronts are all exactly passing through the axial location of the pupils and then focusing at an image point on the image plane. However, due to a defect of optical components and image systems, the wavefront cannot follow the exact path of the CR and maintain perfect spherical shape during transition. Thus, a perfect imaging point cannot be formed because the aberration has been introduced by the defect.

From Fig. 4, it is evident that a shift occurs away from the ideal Gaussian image point $P^*_1$ to a real image point $P_1$. The reason of the shift is because the imaging system’s aberrations induce an OPD (optical path difference). If the whole optical system is treated as a linear system, the OPD should be the difference between the optical path of $P_0P^*_1$ and $P_0P_1$. Since no aberrations are introduced in the object space, the optical paths between them in this section are the same. Thus, the front half of the system can be neglected. One only needs to consider the section from plane of EP to the image plane. This section has been shown in Fig. 5 [26].
Figure 5: Latter half portion of optical system [26].
In Fig. 5, $O_1$ and $O'_1$ represent the intersections of the optical axis (z-axis) with the image plane and EP plane respectively. The points $Q_{AW}$ and $Q_{RW}$ are the intersections of the ray $P_1'P_1$ with the AW (aberrant wavefront) and RW (reference wavefront) respectively. Here, the reference wavefront is actually a Gaussian reference spherical wavefront. The optical path difference of the aberration wavefront with respect to the Gaussian spherical wavefront is a way of describing the aberrations of the optical components being tested. In order to distinguish the previous polynomial $W(x, y)$, the symbol $\Phi$ is used here to represent the optical path length and brackets $[\ ]$ denote the two ends of the optical length but the functionality of both is actually the same. Based on above rule, the following equation is obtained [26]:

$$\Phi = [Q_{AW}Q_{RW}] \quad (5)$$

This optical path length may be called an aberration of the wave at $Q_{RW}$ point or just a wave aberration for simplicity.

Observing the optical path in Fig. 4 and Fig. 5, it is not difficult to use other equations to replace the above one by using geometric optics. Along the direction of the beam, both points $Q_{AW}$ and $Q_{RW}$ originate from starting point $P_0$, and thus the Eq. 5 can be re written as [26]

$$\Phi = [P_0Q_{RW}] - [P_0Q_{AW}] \quad (6)$$
For this situation, the wavefront AW is considered to pass through the center of the EP, which means it will exactly coincide with the wavefront RW in the absence of aberrations. Since all points on the same wavefront have the same phase information, center point \( O'_1 \) has an equivalent phase of point \( Q_{AW} \), and thus \([P_0Q_{AW}]\) can be replaced with \([P_0O'_1]\). Then, Eq. 6 is expressed in this way:

\[
\Phi = [P_0Q_{RW}] - [P_0O'_1] \quad (7)
\]

Because two sets of mutually parallel Cartesian coordinates at \( O_0 \) and \( O_1 \) are located along the optical axis of the system at object and image spaces respectively, there must exist a relationship of points in both spaces. Utilizing expressions for the wave aberration in terms of Hamilton’s point characteristic function of the system, the optical path length, such as \([P_0Q_{RW}]\), of the ray between two points is considered as a function of their coordinates[27].

\[
[P_0Q_{RW}] = V(x_0, y_0, z_0; x, y, z) = \int_{P_0}^{Q_{RW}} n \, ds \quad (8)
\]

According to the coordinates in the Fig. 5, points \( O_0 \) and \( O_1 \) are regarded as original points in object and image spaces, respectively. Therefore the point \( O'_1 \) in the image space is distance of \(-D_1\) away from the original point \( O_1 \), and \( z_0 \) is equal to zero. Using these points’ coordinates, Eq. 7 is rewritten as:
\[
\Phi = (x_0, y_0, 0; x, y, z) - V(x_0, y_0, 0; 0, 0, -D)
\]  

This expression is complicated for further calculation because there are five variables in one function. One of variables may be canceled out or replaced using a relation between them. The coordinates \((x, y, z)\) of point \(Q_{RW}\), which are not independent, can establish a relation to the coordinates in the object plane by using the radius of curvature of reference sphere \(R\).

\[
(x - x_1^*)^2 + (y - y_1^*)^2 + z^2 = R^2
\]  

where \(x_1^*\) and \(y_1^*\) are the coordinates of Gaussian imaging point \(P_{1}^*\) of wavefront RW at image plane. According to the Gaussian lateral magnification \(M = \frac{h'}{h}\) (\(h'\) and \(h\) are the height of object point and Gaussian image point from optical axis, respectively), one can define a relationship as follows:

\[
\begin{align*}
x_1^* &= Mx_0; \\
y_1^* &= My_0;
\end{align*}
\]  

According to \textbf{Fig. 5}, the radius of wavefront \(RW\) can be expressed as:
Substituting Eq. 11 and Eq. 12 into Eq. 10, then variable $z$ can be rewritten with respect to four variables, as follow:

$$R = \left(x_1^2 + y_1^2 + D_1^2\right)^{1/2}$$  \hspace{1cm} (12)

Therefore, the variable $z$ can be replaced, so that optical path length function of the system $\Phi$ can be regarded as a function of $x_0$, $y_0$, $x$ and $y$ only [27]

$$z = \sqrt{D_1^2 + 2M(xx_0 + yy_0) - (x^2 + y^2)}$$  \hspace{1cm} (13)

According to the connection of the ray aberrations and the wave aberrations derived by J. L. Rayces [28], the mathematical relationship can be expressed in this way:

$$\partial x = x_1 - x_1^* = -\frac{R'}{n'} \cdot \frac{\partial \Phi}{\partial x}$$

$$\partial y = y_1 - y_1^* = -\frac{R'}{n'} \cdot \frac{\partial \Phi}{\partial x}$$  \hspace{1cm} (15)
where \( R' \) is the radius of curvature of wavefront AW (here \( R' = \left[ (x_1 - x)^2 + (y_1 - y)^2 + z^2 \right]^{1/2} \)), and \( n' \) is the refractive index of image space (almost equal to 1).

The above relationship has been shown in Fig. 6 [29].

Figure 6: The relation between wavefront aberration and ray aberration [29].
In Fig. 6, since the ray aberration $\partial x$ and $\partial y$ only depend on the coordinates of the system and the system is rotationally symmetrical about the optical axis, the ray aberration must sustain invariant no matter how the angle in the system has been twisted about the optical axis. Based on this concept, the wave aberration also will not be changed during the rotation, unless the value of $R'$ changes. Therefore, the only factor balancing the relationship of ray aberrations and wave aberrations is the radius $R'$, which is dependent on the coordinates of the point $P_1$. However, because of the aberration, one cannot say that the Gaussian lateral magnification exactly fits the object point $P_0$ and real image point $P_1$; there still can exist a certain proportion between them, like $x_1 = Ax_0$ (A is an hypothetical coefficient). It is apparent, based on all relations and assumptions given above, that the aberration $\Phi$ depends on the four variables $(x_0, y_0; x, y)$ only through the three combinations: $x_0^2 + y_0^2$, $x^2 + y^2$ and $x_0x + y_0y$ [25].

To simplify the aberration function and consider all points under the test area, the aberration function can be expressed by a power series in terms of coordinates of the object and the pupil points [25]

$$\Phi(x_0, y_0; x, y) = a_{j k l m} \sum_{j=0}^{\infty} x_0^j \sum_{k=0}^{\infty} y_0^k \sum_{l=0}^{\infty} x^l \sum_{m=0}^{\infty} y^m$$ (16)
At this point, replacing the Cartesian coordinate with polar coordinates is the best way of presenting the exact operation of the system at the next section because the test wavefront emerging from the optical system always keeps a cylindrical path. Using a polar coordinate system can effectively emphasize the property of rotational symmetry in the optical system. By tracing the wavefront at the object plane and image plane, the corresponding coordinates have been shown in Fig. 7.

![Figure 7: Cartesian coordinate converts into polar coordinate at object and exit pupil planes.](image)

In the Fig. 7, $h$ and $r$ are the height of any arbitrary point on the object plane and image plane from the original point on the optical axis, and $\theta_0$ and $\theta$ are the angles...
with respect to $x_0$-axis and $x$-axis, respectively. The translation between them is written as:

\[
\begin{aligned}
x_0 &= h \cos \theta_0 \\
y_0 &= h \sin \theta_0 \\
x &= r \cos \theta \\
y &= r \sin \theta
\end{aligned}
\]  

(17)

Substituting them into power-series expression \textbf{Eq. 16} [25]:

\[
\Phi(h, r, \theta - \theta_0) = \sum_{l=0}^{\infty} \sum_{p=0}^{\infty} \sum_{m=0}^{\infty} c_{lpm} (h^2)^l (r^2)^p [hr \cos(\theta - \theta_0)]^m
\]

\[
= \sum_{l=0}^{\infty} \sum_{p=0}^{\infty} \sum_{m=0}^{\infty} c_{lpm} h^{2l+m} r^{2p+m} \cos^m(\theta - \theta_0)
\]

(18)

where $c_{lpm}$ is the expression coefficient, and $l, p$ \textit{and} $m$ are the positive integers as well as they indicate the order of each variable. For this equation, the degree of each term of the power series is defined as the sum of orders of variables in the object and pupil coordinates ($2l + 2p + 2m$). It is evident that the degree of each term is always even. Regarding this equation, the degree of the terms in this power series can be any value because each value of $l, p$ \textit{and} $m$ ranges from zero to infinity. However, in reality, the value of the degree for each term is constrained in a certain range according to acceptable degree of terms.
By figuring out the acceptable degree of terms, the power series expansion can be simplified further. Thus, take a look at any term, of which \( 2p + m = 0 \). It means that the exponent of variable \( r \) is equal to zero so that these terms do not depend on \( r \). However, it is a paradox that those terms are independent of \( r \) must end up at zero since the aberration associated with the chief ray \( (r = 0) \) is zero. Therefore, zero-degree terms should not be considered here, such as \( c_{300}h^6, c_{400}h^8 \), etc. In other word, \( p \) and \( m \) cannot be zero at same time, and therefore one must be non-zero thus \( 2p + m \geq 1 \).

The terms of second degree are also abandoned here. Due to the above inequality regarding \( p \) and \( m \), only two cases need to be considered one \( p = 1 \); the other \( m = 0 \). In the first case, the term \( c_{010}r^2 \) represents a defocus aberration which is independent of \( h \) but such aberration can be eliminated by adjusting the image receiver in a slightly different plane, like shifting the image plane in the longitudinal direction. Thus, it is apparent that this term should be zero since wave aberration of aberrant image point with respect to the Gaussian image point goes against which the aberration function is defined. Similarly, in the second case, the term \( c_{001}hr \cos(\theta - \theta_0) \) should be zero because it represents a wavefront tilt aberration which can be corrected by a transverse shift of the image receiver. Therefore, the terms of second degree turn out to be unacceptable.

Hence, the subscript of the power series expansion of aberration function is comprised of 4, 6, 8, etc. and the corresponding aberrations are referred to as primary,
secondary, tertiary aberration, etc. In order to simplify and meet the conditions discussed above, the equation should be adjusted [25]

\[
\Phi\left(\bar{h}, \bar{r}, \theta - \theta_0\right) = \sum_{l=0}^{\infty} \sum_{n=1}^{\infty} \sum_{m=0}^{n} 2^{l+m} a_{nm} h' r^{2l+m} \cos^n(\theta - \theta_0)
\] (19)

where \(2^{l+m} a_{nm}\) are the new coefficients for expansion terms, \(n\) is a symbol representing \(2p + m\), of which value is starting from one, and \(h\) has been replaced with Gaussian image point’s height \(h'\).

If the aberration terms having different dependence on coordinates in object space yet the same dependence on coordinates on pupil are combined so that there is only one term for each pair of \((n, m)\) values then, the Eq. 19 will be rewritten as:

\[
\Phi(\rho, \theta - \theta_0) = \sum_{n=1}^{\infty} \sum_{m=0}^{n} a_{nm} \rho^n \cos^m(\theta - \theta_0)
\] (20)

where \(\rho = \frac{r}{t}\) is radial variable normalized by the radius \(t\) of the exit pupil, and the expansion coefficient is

\[
a_{nm} = a^n \sum_{l=0}^{\infty} 2^{l+m} a_{nm} h' r^{2l+m}
\] (21)
Expanding the cosine function in above equation, the results are approximately:

\[ \cos^m(\theta - \theta_0) \approx \cos^m \theta \cos^m \theta_0 + \sin^m \theta \sin^m \theta_0 \]  \hspace{1cm} (22)

Because angle \( \theta_0 \) is an arbitrary original angle with any degree which is set to be constant, the values of \( \cos^m \theta_0 \) and \( \sin^m \theta_0 \) are treated as constant value as well. Therefore Eq. 20 can be rewritten as:

\[ \Phi(\rho, \theta) = \sum_{n=1}^{\infty} \sum_{m=0}^{n} \rho^n (A_{nm} \cos^m \theta + B_{nm} \sin^m \theta) \]  \hspace{1cm} (23)

where:

\[ A_{nm} = a^n \sum_{l=0}^{\infty} 2^{l+m} a_{nm} h^{2l+m} \cos^m \theta_0 \]

\[ B_{nm} = a^n \sum_{l=0}^{\infty} 2^{l+m} a_{nm} h^{2l+m} \sin^m \theta_0 \]

By applying the relation of coefficients of power series and Zernike-Polynomial expression, the cosine and sine functions in Eq. 23 can be switched into their correspondingly approximate form, and then substitute into Eq. 23 which results in [30]
This developed equation is the key function for representing the actual wavefront going through the whole optical system. Now applying this equation and considering the mechanism of the rotational interferometer, the original wavefront and the one duplicated and rotated with respect to the original one by an amount of $\varphi$ should be expressed as:

\[ \text{The original wavefront} = \Phi_1(\rho, \theta) \]
\[ \text{The duplicated wavefront} = \Phi_2(\rho, \theta + \varphi) \]  

In order to develop the following equation easily, one can assumes that the rotation angle $\varphi$ is divided into two equal parts and distributed to each wavefront separately.

\[ \text{The original wavefront} = \Phi_1\left(\rho, \theta - \frac{\varphi}{2}\right) \]
\[ \text{The duplicated wavefront} = \Phi_2\left(\rho, \theta + \frac{\varphi}{2}\right) \]

It is easy to obtain the optical path difference between these two wavefronts by subtracting one from the other, resulting in [22]

\[ \Delta\Phi(\rho, \theta) = \Phi_1\left(\rho, \theta - \frac{\varphi}{2}\right) - \Phi_2\left(\rho, \theta + \frac{\varphi}{2}\right) \]
Based on Eq. 24, the equation of optical path difference between them can be expanded in terms of power series [1]

$$\Delta \Phi(\rho, \theta) = \sum_{n=1}^{\infty} \sum_{m=0}^{n} \rho^n \left[ A_{nm} \cos m \left( \theta - \frac{\varphi}{2} \right) + B_{nm} \sin m \left( \theta - \frac{\varphi}{2} \right) \right]$$

$$- \rho^n \left[ A_{nm} \cos m \left( \theta + \frac{\varphi}{2} \right) + B_{nm} \sin m \left( \theta + \frac{\varphi}{2} \right) \right]$$

$$= \sum_{n=1}^{\infty} \sum_{m=0}^{n} \rho^n \left[ A_{nm} \left[ \cos m \left( \theta - \frac{\varphi}{2} \right) + \cos m \left( \theta + \frac{\varphi}{2} \right) \right] \right.$$  

$$- B_{nm} \left[ \sin m \left( \theta - \frac{\varphi}{2} \right) + \sin m \left( \theta + \frac{\varphi}{2} \right) \right] \right)$$

(29)

Because the inversion shear interferometer in this thesis is an axially symmetric optical system, the aberrations, which are also symmetric about the optical axis, must be canceled out, and thus the coefficient $B_{nm}$ becomes zero. One also can obtain the same conclusion by observing Eq. 29 that the one side boundary of the angle $\theta$ coincides with the coordinate $y$ in the $x$-$y$ plane (the other is on the $x$-axis). In Fig. 8, due to the symmetry about $y$-axis of sine function, the coefficient $B_{nm}$ is eliminated. Then Eq. 29 is rewritten as [1]:
Figure 8: The properties of sine and cosine functions.
\[ \sum_{n=1}^{\infty} \sum_{m=0}^{n} \rho^n A_{nm} \left[ \cos m \left( \theta - \frac{\varphi}{2} \right) + \cos m \left( \theta + \frac{\varphi}{2} \right) \right] = \sum_{n=1}^{\infty} \sum_{m=0}^{n} 2 \rho^n A_{nm} \sin m \theta \sin \frac{m \varphi}{2} \]  (30)

By considering the acceptable subscript of \( A \) representing the kind of aberration, only two primary aberrations are contained in this expression, one astigmatism \((n = 2, m = 2)\) and the other one coma \((n = 3, m = 1)\). In addition, a tilt aberration \((n = 1, m = 1)\) about \( x \)-axis is also present, but one may ignore this aberration since its effect is very small compared to the other two. Thus, Eq. 30 can be expressed in terms of these two primary aberrations:

\[ \Delta \Phi(\rho, \theta) = 2A_{22} \rho^2 \sin 2\theta \sin \varphi + 2A_{31} \rho^3 \sin \theta \sin \frac{\varphi}{2} \]  (31)

which can also be written as

\[ \Delta \Phi(\rho, \theta) = 2A_{22} \rho^2 \sin 2 \left( \theta + \frac{\varphi}{4} \right) \sin \varphi + 2A_{31} \rho^3 \sin \left( \theta + \frac{\varphi}{2} \right) \sin \frac{\varphi}{2} \]  (32)
Observing this expression and referring to the concept developed by Murty and Hagerott [31], the sensitivity of the optical system for astigmatism and coma is changing along with rotation angle $\varphi$.

$$\sigma_{ast} = 2 \sin \varphi$$  \hspace{1cm} (33)

where $\sigma_{ast}$ represents the sensitivity for astigmatism

$$\sigma_{coma} = 2 \sin \frac{\varphi}{2}$$  \hspace{1cm} (34)

where $\sigma_{coma}$ represents the sensitivity for coma. In the two expressions above, the sensitivities for astigmatism and coma are changing along with the change of rotational angle $\varphi$. It is apparent that the period of the sensitivity of coma is as twice long as that of astigmatism, which relation has been shown in Fig. 9 [1]. According to Fig. 9, as the rotational angle $\varphi$ increases to $90^\circ$, the sensitivity for astigmatism reaches a maximum point while coma does not. However, the desirable outcome is that when rotational angle $\varphi$ reaches max angle of $180^\circ$, the sensitivity for coma approaches the peak and removes the effect of astigmatism at the same time. In addition, the relative sensitivity at that point is twice as high in comparison, which is a significant result from this thesis. Thus, the coma aberration can be exactly extracted from dominant aberrations by enhancing its sensitivity by requiring that the test wavefront interfere with its reversed one.
Figure 9: The relative sensitivities of rotational shear interferometer for astigmatism and coma with respect to angle [1].

2.2 Fast quantitative analysis

The analysis of large amounts of data cannot be accomplished by using the traditional method of finding the centers of the fringes and analyzing data on a regular grid. The traditional method of analysis has been gradually eliminated since the use of computers became prevalent. For image data analysis in this thesis, the CCD contains $1600 \times 1600$ pixels (i.e. data points) that need to be processed. Spatial Fourier
transform is one of the novel and effective methods of specifically handling such large data analysis.

The concept behind spatial Fourier transform methodology is that the target information riding on the information carrier (the carrier is actually straight interference fringes with almost constant frequency) is extracted via applying a series of 2-D Fourier transforms followed by inverse Fourier transforms on the target interference pattern, the principle of which is similar to a Moire interferometer [32]. The phase information can then be obtained directly. Combining phase unwarping and curve fitting techniques, precise values of primary aberrations can also be obtained easily.

Any straight interference fringes with regular frequency, like grating, can be treated as an information carrier; however, the problem is that one cannot simply place an exterior gratings on the test image. In other words, the interference fringes come from the target wavefront itself. Based on this point, lateral shear interferometer or inversion shear interferometer is an easy method for duplicating the target wavefront. In addition, using holographic components for shearing, it is even easier to generate a high frequency information carrier.
A basic setup of lateral shear interferometer uses two identical holographic plates (shown as Fig. 10) to create lateral shearing and tilt. When a nearly collimated light with aberrations falls on the two holographic plates, two identical target wavefronts are emitting behind the plates. The only difference between them is that the waves are traveling with slightly two different angles. Therefore the lateral shear is based upon the amount of frequency difference between these two holographic plates.

Let us assume that the two holographic lens are displaced by the amount of $\Delta x$. The same amount of the shear will occur at the focal point plane. The holo-lens has a
property similar to the convex lens, having a certain focal length decided during fabrication. After these two focal points, the beams diverge, and they interfere with each other at the overlapping area. Thus, according to this basic schematic, one can see that obtaining interference patterns is the same as Young’s double slit experiment. By considering the effect of Young’s double slit experiment and the setup here, the interference fringes are straight (perpendicular to the line joining the two foci, and parallel to one another with a nearly constant period). The fringes’ frequency ν can also be expressed in the form of Young’s double slits

\[
\nu = \frac{\Delta x}{\lambda \cdot L}
\]  

(35)

where \( \lambda \) is the wavelength used in this experiment, and \( L \) is the distance from the focal point of the holographic plate to the CCD camera.

After the two wavefronts passes through the focal plane of the holographic plate, the phase of the two wavefronts can be separately expressed as [33]

\[
\phi_1(x, y) = k w(x, y) + \frac{k[x^2+y^2]}{f}
\]  

(36)

\[
\phi_2(x, y) = k w(x + \Delta x, y) + \frac{k[(x + \Delta x)^2 + y^2]}{f}
\]  

(37)
where $k$ is propagation constant, $w(x, y)$ is the optical path function (it can be treated as the aforementioned optical path function in the last section), and $f$ is the focal length of the holographic lens.

When the two wavefronts meet at the plane of the CCD camera, the interference pattern is modulated by the phase difference between them, which can be expressed as

$$
\Delta \phi = \phi_2(x, y) - \phi_1(x, y) = k \frac{\partial w}{\partial x} \Delta x + \frac{2k x \Delta x}{f}
$$

(38)

Substituting frequency from equation Eq. 35 into the above equation, the final equation becomes [33]

$$
\Delta \phi = k \frac{\partial w}{\partial x} \Delta x + 2\pi vx
$$

(39)

The interference pattern is modulated by the phase difference. By applying the phase difference, the general expression for an interference pattern can be expressed as $f(x, y)$ [33]

$$
f(x, y) = I(x, y) \left\{ 1 + M \exp \left( i \left[ k \frac{\partial w}{\partial x} \Delta x + 2\pi vx \right] n \right) \right\}
$$

(40)
\[ I(x, y) \] is the average intensity of the background, \( M \) is the modulation of the fringes, and \( n \) is the order number. The target information of the wavefront is contained in the term \( k \frac{\partial w}{\partial x} \Delta x \). In order to extract out this information, spatial Fourier transform is used. Due to the shear \( \Delta x \) along the x direction, the Fourier transform is applied to \textbf{Eq. 40} in the x direction and can be expressed as [33]

\[
G(v_x, y) = \int_{-\infty}^{\infty} f(x, y) \exp(-i 2\pi v x) dx
\] (41)

\( G(v_x, y) \) represents the Fourier transform of function \( f(x, y) \) in the frequency domain. By converting the above integral function to a discrete function, Eq. 41 is rewritten as [33]:

\[
G(v_x, y) = A + \sum_{n=-\infty}^{n=+2} [C(v_x - n\nu) + C^*(v_x + n\nu)]
\] (42)

\( A \) represents the central spot in the light spectrum in the frequency domain, \( C \) and \( C^* \) represent the right and left side order functions around the zero order \( A \), and \( n \) is the central frequency of the side orders. As \( n=2 \), \( 2\nu \) represents the central frequency of the second order. Also, terms \( A \), \( C \) and \( C^* \) have their own expression, which are respectively [33]

\[
A = F[I(x, y)]
\] (43)
\[ C(\nu_x - n\nu) = F\{I(x, y) \frac{M}{2} \exp \left( i \left[ k \frac{\partial w}{\partial x} \Delta x - 2\pi \nu \Delta x \right] n \right) \} \] (44)

\[ C^*(\nu_x + n\nu) = F\{I(x, y) \frac{M}{2} \exp \left( i \left[ k \frac{\partial w}{\partial x} \Delta x + 2\pi \nu \Delta x \right] n \right) \} \] (45)

\( F\{\ldots\} \) represents the Fourier transform operator. From these three expressions, it is apparent that the target phase information is contained in the side orders of the spectrum. Thus, primary attention should be focused on side orders at this time. The frequency of the fringes is proportional to the interval between side orders. For instance, high frequency results in long interval between side orders. Therefore, using a high frequency information carrier, the interval is too long to see the second order, thus the first order has been used here.

From the above discussion, it is easy to determine the location of the first order with frequency \( \nu \) (n=1). Then, the first order is moved towards the location of the zero order by the amount \( \nu \), which means that the carrier frequency signal is being removed, i.e. function \( C(\nu_x - \nu) \) changes into [33]

\[ C(\nu_x - \nu) = F\{I(x, y) \frac{M}{2} \exp \left( i \left[ k \frac{\partial w}{\partial x} \Delta x \right] \right) \} \] (46)
In this method, the first order is filtered with an appropriate band pass filter and is then shifted to the center of the spectrum. When an inverse Fourier transform is applied to the centered first order, its inverse Fourier transform can be expressed as

\[ \mathcal{F}^{-1} (C(u_x - nu)) = 1(x, y) M_1 M \exp \left( i k \frac{\partial w}{\partial x} \Delta x \right) \]

Equation (47)

F\(^{-1}\)\((\ldots)\) is the inverse Fourier transform operator, \(M_1\) is a constant introduced due to the transformation. \(k \frac{\partial w}{\partial x} \Delta x\), which contains phase information, is kept constant. By taking the ratio of the real value to the imaginary value in the complex term, one can obtain the phase value which is set to vary between 0 and \(2\pi\). Thus, the phase variance can finally be expressed as [33]

\[ \frac{\partial w}{\partial x} = \frac{mx}{n \Delta x} \]

Equation (48)

where \(m\) is an integer and \(\frac{\partial w}{\partial x}\) is related to the sensitivity. From this equation, it is easy to see that sensitivity is proportional to the number of the order \(n\) and inversely proportional to the wavelength used in this experiment. Therefore, the sensitivity of the novel lateral shear interferometer in this thesis can be controlled by three factors: the rotational angle, the number of diffraction order in the spectrum, and the wavelength used in the experiment.
Through all aforementioned processes, a phase map modulated by $2\pi$ is present. Applying a phase unwrapping technique on the phase map gives a very smooth surface of the wavefront in 3-D. Due to effect of aberrations, the shape is no longer a plane wavefront. At this point, using a curve fitting tool and a primary aberration polynomial the corresponding aberration coefficients can be obtained.
3. EXPERIMENT

3.1 Experimental description

The schematic arrangement of the inversion shear interferometer using spatial Fourier transform is shown in Fig. 11 and Fig. 12. In this figure, it can be seen that the inversion shear interferometer has two main holographic components, a holo-lens and a holo-grating. In order to keep the system in perfect alignment for the second holographic combination and to achieve the objective of the whole system, both the holo-lens and holo-grating in the first combination part should have the same diffraction angle of the first order, but a different diffraction angle between the holo-lens and holo-grating in the second combination. As the holo-grating of the first combination is placed in the path of the collimated light, its first two diffracted orders are collimated. But, the situation in the holo-lens is different; the two first order beams diffracted by the holo-lens in the first combination are convergent and divergent respectively. After the focal point of the first order beam of the holo-lens, a convergent beam is converted into the divergent beam, and all the points on its wavefront will be inversed with respect to the center. In this process, the directions of the convergent beam and collimated beam are coincidence.
Figure 11: The setup of novel inversion shear interferometer.
Figure 12: Photo of the setup.
In the second combination, the previous two diffracted beams act as reconstruction beams. When these two reconstruction beams hit on the two elements, their diffraction beams include two collimated beams with a certain angle between them, which is set during the fabrication process.

In fabrication of holographic optical components, three small yet critical factors need to be determined: angle relation between incident and diffraction beams, focal length of the two holo-lenses, and the diffraction efficiency of every single holographic optical component used in this experiment. They will be the deciding factors in the success of the experiment.

Considering the aforementioned characteristics of compaction and feasibility, straight line shape of arrangement makes the optical system clear and applicable. Here, straight line means that incident and emergent light of the system should be kept parallel. Therefore, the relationship of the incident angle and diffracted angle becomes very important for each holographic optical component. Actually, such a relationship has already been determined in its fabrication. Holographic lenses are fabricated by recording the interference of reference beams and object beams on a holographic plate (shown in Fig. 13).
Figure 13: The process of recording holo-grating and holo-lens.
Figure 14: The process of reconstructing object beams.
Either reference beam or object beam can act as a reconstruction beam. Take the reference beam as an example. When reference beam is used as a reconstruction beam to illuminate the processed holographic plate, the object beam is reconstructed behind the plate (this schematic has been shown in Fig. 14). The interference pattern recorded on holographic plate can be expressed as [34]

\[
I = |U_r|^2 + |U_o|^2 + U_r^* U_o + U_r U_o^*
\]  

(49)

\(U_r, U_o, U_r^*\) and \(U_o^*\) are the reference beam, object beam, conjugate of reference beam, and conjugate of object beam respectively. The previous example of process reconstruction also can be expressed as:

\[
I \cdot t \cdot U_r = (|U_r|^2 + |U_o|^2) t \cdot U_r + t \cdot |U_o|^2 U_o + t \cdot U_r U_r^* \cdot U_o^*
\]  

(50)

where \(t\) is the transmittance. It is apparent to see that the phase information of reconstructed object beam \(t \cdot |U_o|^2 U_o\) does not change its intensity along with transmittance \(t\). In other words, the diffraction angle is the angle at which the two beams interfere in fabrication. As for the quantity of the angle, it depends on the requirement of configuration in the system and diffraction efficiency, which will be discussed below.
Figure 15: The setup of making holographic lens.
The focal length of the holo-lens is another important factor which can give rise to a compact system because the distance between the two parts is doubled its focal length. The focal length is part of the spatial phase information, thus it also can be recorded on the holo-lens. In other words, during fabrication the focal length is selected by choosing the location of the spherical wave (object) source (shown in Fig. 15). The spatial filter, which generates spherical waves, is located 10 centimeters away from the location of holographic plate, and therefore the focal length is 10 centimeters.

The final point that cannot be ignored is the diffraction efficiency. In addition to other regular optical components in this experiment, four holographic lenses are required and only one side diffraction orders are utilized, so that diffraction efficiency has to be high enough for the final image of the interference pattern to be detected both by the CCD camera and the human eye for alignment. In fabrication, diffraction efficiency can be affected by several ways: intensities of the reference and object beams, exposure time, developing and bleaching process, and the selection of diffraction order.

The diffraction efficiency $\eta$ is ratio of intensity of the diffracted beam to the intensity of the incident beam, which can generally be expressed as:

$$\eta = \frac{I_d}{I}$$

(51)
where \( I_d \) is intensity of the diffraction beam, and \( I \) is intensity of incident beam. However, diffraction efficiency can also be expressed in terms of visibility \( V \)

\[
\eta = \frac{1}{4} t_a^2 V^2
\]  

(52)

where \( t_a \) is bias transmittance, and the range of value of visibility \( V \) is from zero to 1, depending on the relative ratio of the two recording beams. Therefore, to make the diffraction efficiency approach its maximum value possible, the value of visibility \( V \) should be close to one. According to Eq. 53 of visibility, only when reference beam and object beam have the same intensity will the visibility be equal to one.

\[
V = \frac{2\sqrt{I_r I_o}}{I_r + I_o}
\]  

(53)

As stated previously, the second factor is exposure time, which strongly depends on what kind of holographic plate is used. In this experiment, the holographic plate is type of PFG-01 from Slavich with related properties shown in Table 1 and Fig. 16 [36]. By observing the chart and figure, the practicable diffraction efficiency of the holographic plate is not lower than 35%, and its maximum is approximately equal to 48% when the exposure energy reaches 115 \( \mu J/cm^2 \). In the experiment of fabricating holographic lens, the area of the detector is 1.33\( cm^2 \), the power of the combined interfering beams is \( 20 \times 10^{-6} W \) and the power density is 0.15 \( W/m^2 \). Thus, the
exposure time required is 7.7 seconds. The timer, which controls the exposure time, was set to 8 seconds, thus the maximum of diffraction efficiency of the holographic lens was almost 47% (shown in Fig. 17).

Table 1: Parameters of PFG-01 [36].

<table>
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<tr>
<th>Name of Indices</th>
<th>VRP-M</th>
<th>PFG-01</th>
<th>PFG-03M</th>
<th>PFG-03C</th>
<th>PFG-04</th>
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<td>1.0</td>
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<td>30</td>
<td>2500-3000</td>
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<td>633 nm</td>
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<td>45</td>
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<td>70</td>
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<td>650</td>
<td>680</td>
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<td>Maximum of spectral sensitization, nm</td>
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<td>625±10</td>
<td>635±5</td>
<td>635±5</td>
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<td>Adhesion between emulsion layer and base after chemical photographic processing, conventional classes A-F</td>
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<td>A-C</td>
<td>A-C</td>
<td>A-C</td>
<td>A-C</td>
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<td>7±1</td>
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<td>12-15</td>
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<td>Antihalation protection</td>
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<td>NO</td>
<td>NO</td>
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</table>
Figure 16: The relation between diffraction efficiency and exposed energy [36].

Figure 17: Maximum diffraction efficiency is obtained by adjusting setup.
The final critical factor for obtaining the highest diffraction efficiency is selecting the correct order of the diffraction beam. The transmittance t of a phase hologram formed by bleaching of an amplitude hologram can be written as

\[ t = \sum_{n=-\infty}^{\infty} i^n J_n(M)e^{in\Delta\phi} \]  

(54)

where \( J_n(M) \) is the nth-order Bessel function (shown in Fig. 18), M is the amplitude of the phase delay, n is the function order and \( \Delta\phi \) is the phase difference between the two recording beams. Based on this equation, the diffraction efficiency \( \eta \) can also be derived as

\[ \eta = J_n^2(M) \]  

(55)

Combining this equation and the property of the Bessel function, only when \( n = 1 \) (first order) can the diffraction efficiency reach the highest value. Thus, the first diffraction order beam was chosen for the experiment. At the same time, the hologram is recorded with two beams of equal intensity to guarantee high intensity modulation in a single grating, thus reducing noise grating effects considerably [34].
When a collimated beam falls on the first combination of holo-lens and holo-grating, two first diffracted waves, one converging and the other one plane, emerge. The resultant beam can be expressed as

\begin{align*}
I_1 &= u^2\{2 + \exp(i\varphi_0) \exp[-i\varphi(x,y)] + \exp(-i\varphi_0) \exp[i\varphi(x,y)]\} \\
I_2 &= u^2\{2 + \exp(i\varphi_0) \exp[-ikx \sin \theta_1] + \exp(-i\varphi_0) \exp[ikx \sin \theta_1]\}
\end{align*}

(56)

(57)
where $u$ is the amplitude, $\exp(i\varphi_0)$ is the spatial part of the on-axis reference plane wave, $\exp[i\varphi(x,y)]$ is the spatial part of the off-axis object spherical wave, $\exp[ikx \sin \theta_1]$ is the spatial part of the off-axis object plane wave and $\theta_1$ is the angle between the reference and object waves. When the incident beam, which is nearly a collimated wave and carrying aberrations of $W(x,y)$, illuminate the first combination, the two first diffracted convergent and plane waves emerging from the combination could be express as $A\{\exp[ikW(x,y) + i\varphi(x,y)]\}$ and $A\{\exp[ikW(x,y) + ikx \sin \theta]\}$ respectively. After passing through focal point of the holo-lens, the convergent wave switches into a divergent one, while the plane wave does not change. Therefore, all points on the divergent wave are inverted with respect to the convergent one, and the expression for divergent wave changes to $A\{\exp[ikW(-x,-y) + i\varphi(x,y)]\}$. At twice the focal distance “$2f$” behind the first combination, the second holographic combination is placed. Both divergent and plane waves work as reconstruction waves at this time. In the second combination, the irradiance of the holo-lens remains the same as the previous holo-lens, but the holo-grating has different spatial frequency than the previous one by decreasing the angle $\theta_2$ in the direction of diffraction during the fabrication, so that shear is generated. The second holo-grating is expressed as

$$I_3 = u^2\{2 + \exp(i\varphi_0) \exp[-ikx \sin \theta_2] + \exp(-i\varphi_0) \exp[ikx \sin \theta_2]\} \quad (58)$$

The phase of these two expected emerging waves can be described as $\varnothing_1$ and $\varnothing_2$.
\[ \begin{align*} 
\varphi_1 &= \varphi_0 + kW(-x, -y) \quad (59) \\
\varphi_2 &= \varphi_0 + kW(x, y) + k(x \sin \theta_1 - \sin \theta_2) \quad (60)
\end{align*} \]

However, after the second combination, there exist two other unexpected waves, one converging and one diverging. Therefore a positive lens was used to focus the converging and diverging waves before and after the foci of the two nearly collimated waves. By using a pinhole with an appropriate diameter, the unwanted beams were eliminated. After the pinhole the two collimated waves interfere and produce an interference pattern which can be expressed as

\[ \Delta \varphi = \varphi_1 - \varphi_2 = k(x \sin \theta_2 - \sin \theta_1) + kW(-x, -y) - kW(x, y) \quad (61) \]

The shear \( \Delta x \) is introduced by the different diffraction angles \( \theta_2 - \theta_1 \) between the two beams along the direction of diffraction (x-axis), which can be written approximately for small angle as

\[ \Delta x = L \sin(\theta_2 - \theta_1) \approx L [\sin \theta_2 - \sin \theta_1] \quad (62) \]

where \( L \) is the distance of second combination and the positive lens. The interference pattern will be perpendicular to the direction of diffraction (x-axis). The spacing of the interference fringes will be the same in the entire region of the overlapping beams and modulated by the aberrations \( W(x, y) \). The spatial frequency \( \nu \) of the fringe is
Therefore the phase difference can be rewritten as

\[
u = \Delta x/\lambda L = \frac{\sin \theta_2 - \sin \theta_1}{\lambda}
\]  \hspace{1cm} (63)

The phase difference of the interferogram \( f(x,y) \) on the CCD can be express as

\[
\Delta \phi = k[W(-x,-y) - W(x,y)] + 2\pi\nu x
\]  \hspace{1cm} (64)

The phase difference of the interferogram \( f(x,y) \) on the CCD can be express as

\[
f(x,y) = I(x,y)\{1 + M \exp\{i[k[W(-x,-y) - W(x,y)] + 2\pi\nu x]n\}\}
\]  \hspace{1cm} (65)

where \( I(x,y) \) is the average intensity of the interferogram, \( M \) is the modulation of the fringe and \( n \) represents the order of the diffraction (here \( n=1 \)).
Figure 19: The original interference pattern of novel inversion shear interferometer.

Figure 19 shows the image of the two sheared and inverted beams captured by the CCD camera. This captured image is then 2-D Fourier transformed and the image in the transform phase is shown in Fig. 20.
Apart from the center spot, two first orders are obtained and the interval between orders is equal to frequency \( v \) (shown in Eq. 63). The larger the order one chooses for analysis, the more noise that is going to be introduced, which is another reason why the first order was selected to extract phase information.

Figure 20: The spectrum is Fourier transform of original image.
Figure 21: Filtering and transferring positive first order towards center.

A band pass filter is then used to extract the desired order, which is shown in Fig. 21. The size of the filter should be reasonable so that it fits correctly according to the size of the side order spot. If it is too big, noise can also be introduced. On the contrary, if it is too small, some target phase information is going to be lost. The target order is moved towards the center of the image by the amount of one frequency $v$ (shown in Fig. 21) such that the information of the carrier is eliminated as shown in Eq. 46. Finally, this
processed image is inverse Fourier transformed. The target phase information of the wavefront under test is extracted and is expressed as:

\[ F^{-1}\left(\hat{B}(v - n u)\right) = I(x, y)C_1 M \exp(i[k[W(-x, -y) - W(x, y)]]) \]  \hspace{1cm} (66)

where \( C_1 \) is a constant introduced by the transformation. This equation shows that the target phase information \( W(-x, -y) - W(x, y) \) is in the imaginary part. Through the method of taking the ratio of the real value to the imaginary value in the complex term, the phase map is obtained and can be expressed as

\[ W(-x, -y) - W(x, y) = m \lambda \]  \hspace{1cm} (67)

where \( m \) is an integer. The phase map is shown in \textbf{Fig. 22}. 
To determine if the target order is in the center position, one must observe the number of fringes. When the number of fringes, as shown in Fig. 22, is the minimum amount, then the target order is in the center position. The target order in the exact center means that the information carrier frequency is eliminated completely. If the information carrier frequency is not erased completely, the extra information will be introduced and more fringes will be shown in the final phase map. When the target orders are not moved in the exact center, as shown in Fig. 23, the phase map has more fringes.
The phase map is modulated by $2\pi$, which means that the phase $\phi$ is a saw tooth function and has discontinues at every $2\pi$ change. Drawing a line cross the middle of Figure 22 and plotting its data curve (shown in Fig. 24 and Fig. 25), it is not difficult to see that there is a jump in the data from the bottom to the top when a $2\pi$ change occurs. If $\phi$ is increasing, the slope is positive and vice versa for decreasing phase.
Figure 24: The plot of data of cross line on the phase map.
The final stage of the phase measurement process is phase unwrapping. Phase unwrapping is the technique which makes the phase map smooth and continuous by eliminating $2\pi$ discontinuities. By using the method of extracting quantitative data, the whole process of unwrapping is much faster and more accurate than ever. The computer calculates the data pixel by pixel. When it finds that one pixel’s value jumps suddenly by amount of more than $\pi$, $2\pi$ will be added or subtracted from the next pixel, depending
on its slope. For example, if a sudden jump occurs and the slope is negative, the following data should have $2\pi$ subtracted. The unwrapped phase map is shown in Fig. 26. Select a rectangular area on Fig. 26 and plot it with 3-D version in Fig. 27.

Figure 26: Unwrapped phase map of the test wavefront.
Figure 27: 3-D plot of a portion of wavefront. In this figure, the radians have been converted into wavelength.

3.2 Experimental data

The size of the CCD used for capturing the image is 1600d for pixels with a pixel size of $7.4 \times 7.4 \, \mu m^2$ with a chip size of $36 \times 24 \, mm^2$ (maximum resolution of 135lp/mm along the x-axis). Therefore, the maximum angle $(\theta_2 - \theta_1)$ to create the shear, which could be tolerated by the CCD, can be determined by the resolution of the CCD. Assuming that at least three pixels are required to resolve one fringe the maximum angle is 3.84 degrees.
The angle difference \((\theta_2 - \theta_1)\) used in this experiment is 2 degrees. Therefore, the diffraction angles of the holo-grating in first combination and the two identical holo-lenses are all 20 degrees, but the diffraction angle of the holo-grating in the second combination is 18 degrees. Thus, the two nearly collimated beams emerge from the system with an angle of 2 degrees between them.

To extract quantitative data, 3-D curve fitting was used on the image shown in Fig. 26. 3-D curve fitting applies to the matrix only. Therefore, three different rectangular areas have been selected. After applying 3-D curve fitting, three values of coma have been obtained. They are \(1.775 +/- 4.248 \times 10^{-6}\), \(1.769 +/- 4.233 \times 10^{-6}\), and \(1.802 +/- 4.316 \times 10^{-6}\). The values are very close to each other. The average value of coma measured in this experiment is 1.782. The same test lens was tested in a lateral shear interferometer, which showed that the value of coma was equal to 0.9753 [35]. In the comparing these two results, the value of coma in this experiment is twice the value of coma in the lateral shear interferometer. The result agrees with the expectation of this thesis. The inversion shear interferometer is double sensitive to coma.
4. CONCLUSION

This thesis presents a new inversion shear interferometer and a method to extract phase using a digital process and spatial Fourier Transform. A combination of holo-lens is used for inverting all points of the wavefront with the wavefront generated from a holographic grating. The method also takes advantage of diffraction angle in the second combination to generate lateral shear and the information carrier. Theoretical equations have been derived to demonstrate why an inversion shear interferometer is only sensitive to coma aberration and tilt aberration at a rotation angle of 180 degrees.

This thesis also presented some advantages of the novel inversion shear interferometer. Digital processing gives rise to obtaining phase information faster and making analysis of the data more reliable. The key point is the introduction of the technique of spatial Fourier transform to an inversion shear interferometer with the aid of holographic optical devices.

The novel inversion shear interferometer does however have its own disadvantages. For example, in Fig. 19, there is some obvious noise in the unwrapped phase map. When applying the process of phase unwrapping, noise can cause unwanted jumps in the phase map (shown in Fig. 25). Diffraction efficiency is also another problem. The maximum DE of each piece of holo-optical components is 48%, after passing through two combinations of holo-optical components, the intensity of one
wavefront will be less than 20%. In addition, in this experiment, so many diffraction optical devices have been utilized that a multiple of stray light has been produced, which is also source of noise as well.

However, above issues can be solved by using replacing holo-lens and holo-grating with a volume hologram. Volume hologram has a capability of reaching nearly 100% diffraction efficiency. This means that there is no stray light anymore because 100% diffraction leads to only one diffracted order, even zero order is not present. Volume hologram is strictly subject to Bragg’s theory. If electro-optical material such as LiNbO$_3$ is used, all interference patterns can be recorded in one material so that the process of alignment will be much easier and faster. At the same time, external errors, such as air perturbation, can be completely removed. Moreover, its rewritable characteristic makes the experiment more flexible as well as lowers the experiment cost.
LIST OF REFERENCES


